Gravitational microlensing and non-singular black holes

Jens Boos & Hao Hu
Karlsruhe Institute of Technology, Germany

ITP Seminar
Thursday, July 3, 2025, 1:30pm

In preparation...

Microlensing and photometric lightcurves of non-singular black holes: a ray tracing approach

Jens Boos[*] and Hao Hu[†]
Institute for Theoretical Physics, Karlsruhe Institute of Technology, D-76128 Karlsruhe, Germany
(Dated: July 2, 2025)

We study the gravitational lensing of various static and spherically symmetric non-singular black holes (and horizonless compact objects of similar size). Via ray tracing the lensing problem is treated entirely numerically, allowing the study of a wide class of models. We focus on two cases: (i) the lensing of an accretion disk, and (ii) the lensing of a background object. While the former allows comparisons between black hole shadow sizes and the general morphology of the accretion disk across various black hole models, the latter scenario is used to address the gravitational microlensing of such objects. By assuming proper motion of the gravitational lens, we record the apparent brightness as a function of time, resulting in a photometric light curve. We find that non-singular black hole models tend to decrease the Einstein ring and therefore increase the maximum brightness recorded in microlensing light curves.

1/5 Motivation

There is something very wrong with black holes.

Black holes and singularities

Spherically symmetric and static metric:

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + B(r)\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\varphi^2$$

Invoking the vacuum field equations of general relativity:

$$A(r) = \frac{1}{B(r)} \equiv f(r) = 1 - \frac{2GM}{r}$$

The location where $A(r) = 0$ is called the "event horizon:" a null 3-surface of no return.
The location where $A(r) \to -\infty$ is a spacetime singularity. Need to **resolve** this!

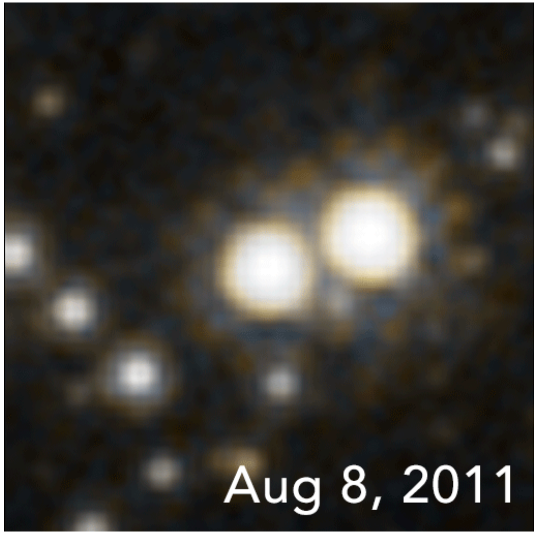
$$A(r) = \frac{1}{B(r)} \equiv f(r) = 1 - \frac{2GM}{r}\frac{r^3}{r^3 + 2GM\ell^2} \quad \sim \quad 1 - \frac{r^2}{\ell^2} \quad \text{for } r \to 0\,.$$

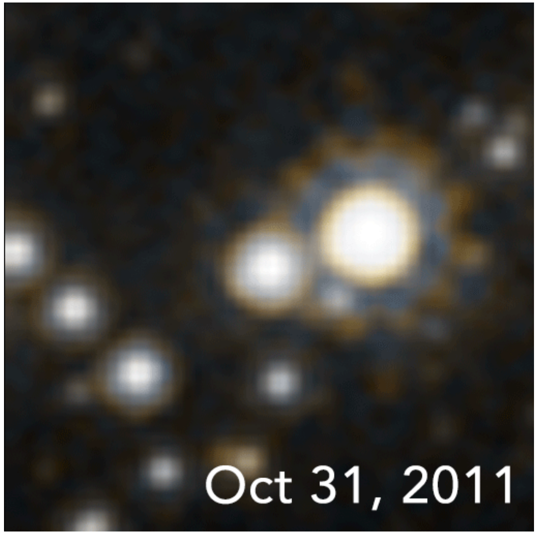
There is a whole "zoo" of non-singular black hole metrics. Can we constrain this?

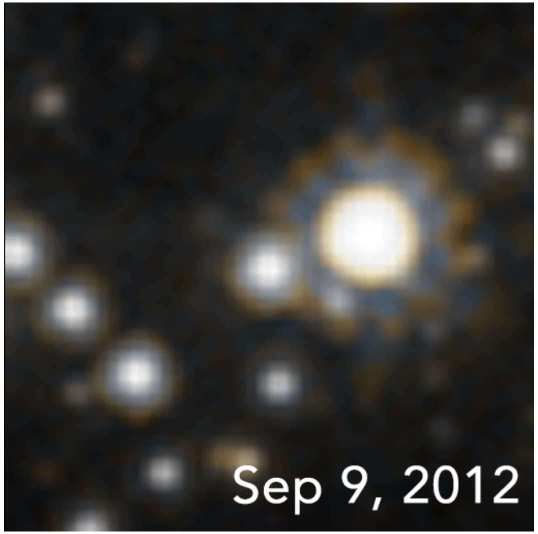
1/5 Motivation

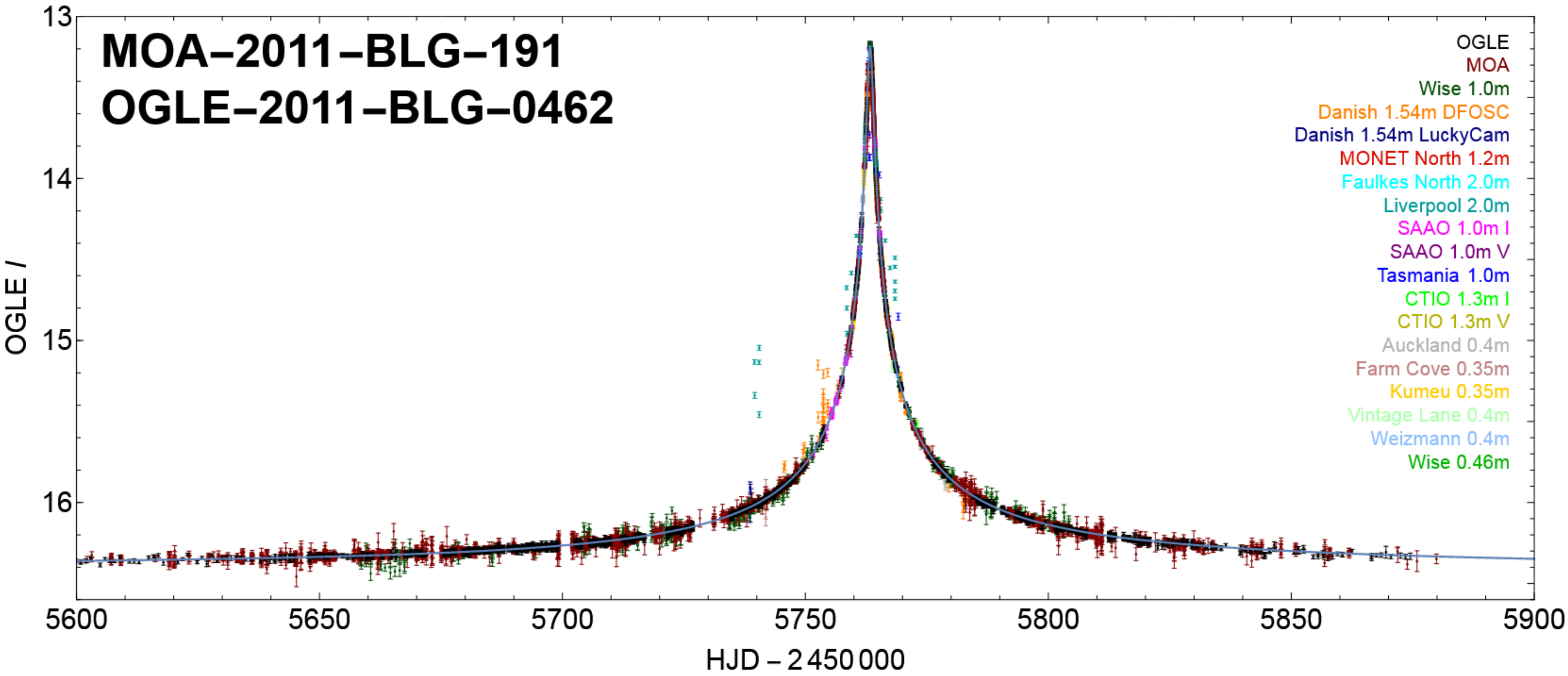
No need for new expensive experiments.

These have been taken with the Hubble Space Telescope





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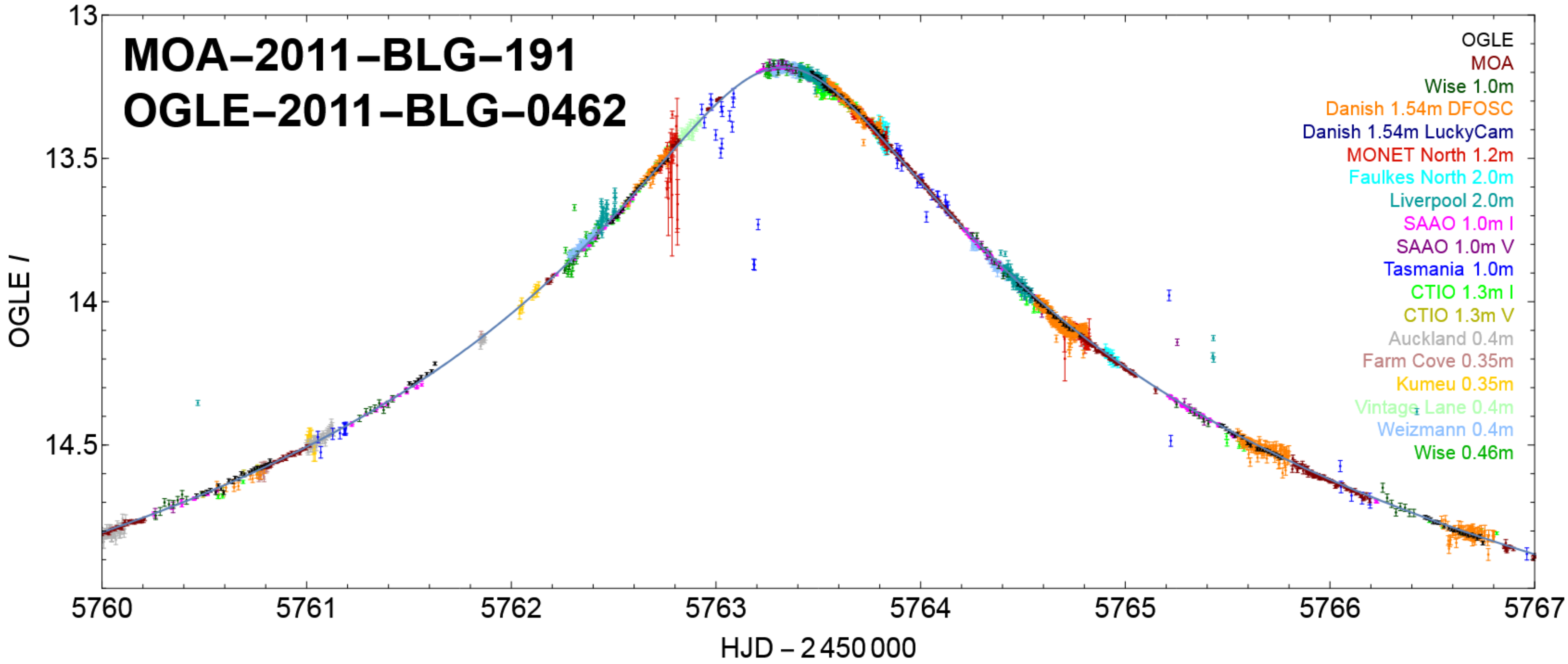
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THE ASTROPHYSICAL JOURNAL, 933:83 (28pp), 2022 July 1 https://doi.org/10.3847/1538-4357/ac739e

An Isolated Stellar-mass Black Hole Detected through Astrometric Microlensing*

Kailash C. Sahu[1,70], Jay Anderson[1], Stefano Casertano[1], Howard E. Bond[1,2], Andrzej Udalski[3,71], Martin Dominik[4,72], Annalisa Calamida[1], Andrea Bellini[1], Thomas M. Brown[1], Marina Rejkuba[5], Varun Bajaj[1], Noé Kains[6,73], Henry C. Ferguson[1], Chris L. Fryer[7], Philip Yock[8], Przemek Mróz[3], Szymon Kozłowski[3], Paweł Pietrukowicz[3], Radek Poleski[3], Jan Skowron[3], Igor Soszyński[3], Michał K. Szymański[3], Krzysztof Ulaczyk[3,9], Łukasz Wyrzykowski[3]
(OGLE Collaboration),
Richard K. Barry[10], David P. Bennett[10,11], Ian A. Bond[12], Yuki Hirao[13], Stela Ishitani Silva[10,14], Iona Kondo[13], Naoki Koshimoto[10], Clément Ranc[15], Nicholas J. Rattenbury[16], Takahiro Sumi[13], Daisuke Suzuki[13], Paul J. Tristram[17], Aikaterini Vandorou[10,11]
(MOA Collaboration),
Jean-Philippe Beaulieu[18,19], Jean-Baptiste Marquette[20], Andrew Cole[18], Pascal Fouqué[21], Kym Hill[18], Stefan Dieters[18], Christian Coutures[19], Dijana Dominis-Prester[22], Clara Bennett[23], Etienne Bachelet[24], John Menzies[25], Michael Albrow[26], Karen Pollard[26]
(PLANET Collaboration),
Andrew Gould[27,28], Jennifer C. Yee[29], William Allen[30], Leonardo A. Almeida[31,32], Grant Christie[33], John Drummond[34,35], Avishay Gal-Yam[36], Evgeny Gorbikov[37], Francisco Jablonski[38], Chung-Uk Lee[39], Dan Maoz[40], Ilan Manulis[41], Jennie McCormick[42], Tim Natusch[33,43], Richard W. Pogge[28], Yossi Shvartzvald[41]
(μFUN Collaboration),
Uffe G. Jørgensen[44], Khalid A. Alsubai[45], Michael I. Andersen[46], Valerio Bozza[47,48], Sebastiano Calchi Novati[49], Martin Burgdorf[50], Tobias C. Hinse[51,52], Markus Hundertmark[15,73], Tim-Oliver Husser[53], Eamonn Kerins[54], Penelope Longa-Peña[55], Luigi Mancini[27,56,57], Matthew Penny[58], Sohrab Rahvar[59], Davide Ricci[60], Sedighe Sajadian[61], Jesper Skottfelt[62], Colin Snodgrass[63,73], John Southworth[64], Jeremy Tregloan-Reed[65], Joachim Wambsganss[15], Olivier Wertz[66]
(MiNDSTEp Consortium),
and
Yiannis Tsapras[15], Rachel A. Street[24], D. M. Bramich[67,68], Keith Horne[4,74], and Iain A. Steele[69]
(RoboNet Collaboration)

* This research is based in part on observations made with the NASA/ESA Hubble Space Telescope, obtained from the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS 5-26555.

2/5 Mathematical background

Geodesics, gravitational lensing.

Null geodesics

Particle motion described by worldline: $x^\mu(\lambda) = (\ t(\lambda),\, r(\lambda),\, \theta(\lambda),\, \varphi(\lambda)\ )$

$$0 = \frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\lambda^2} + \Gamma^\mu{}_{\nu\rho}\frac{\mathrm{d}x^\nu}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\rho}{\mathrm{d}\lambda} \qquad \left\{ \begin{aligned} 0 &= t'' + \frac{f'r't'}{f}\,, \\ 0 &= r'' + \frac{f'(-r'^2 + f^2 t'^2)}{2f} - fr(\theta'^2 + \sin^2\theta\varphi'^2)\,, \\ 0 &= \theta'' - \cos\theta\sin\theta\varphi'^2 + \frac{2r'\theta'}{r}\,, \\ 0 &= \varphi'' + \frac{2(r' + r\theta'\cot\theta)\varphi'}{r} \end{aligned} \right.$$

Null character is enforced via initial condition $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = 0$.

Motion of light is completely integrable for stationary black holes, leading to simplifications.

Null geodesics

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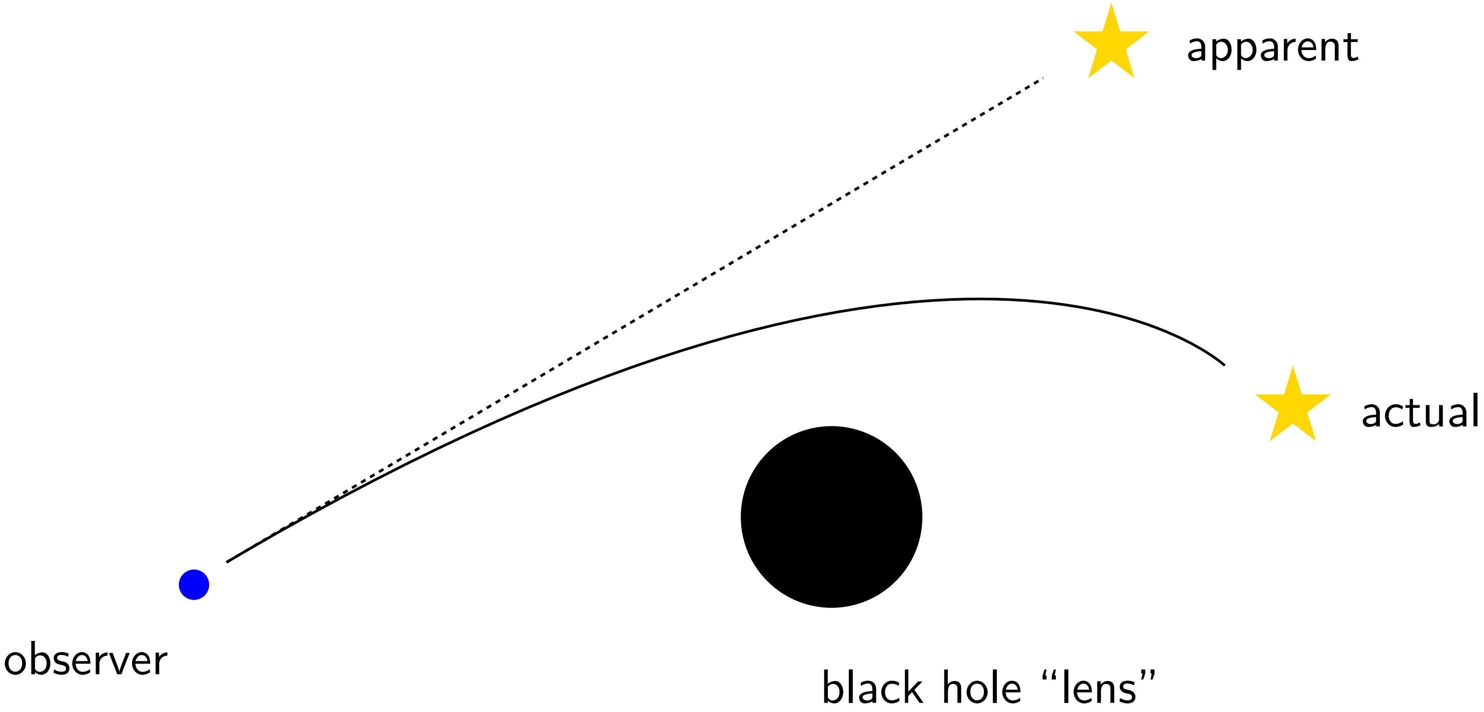
Motion of light is completely integrable for stationary black holes, leading to **simplifications**.

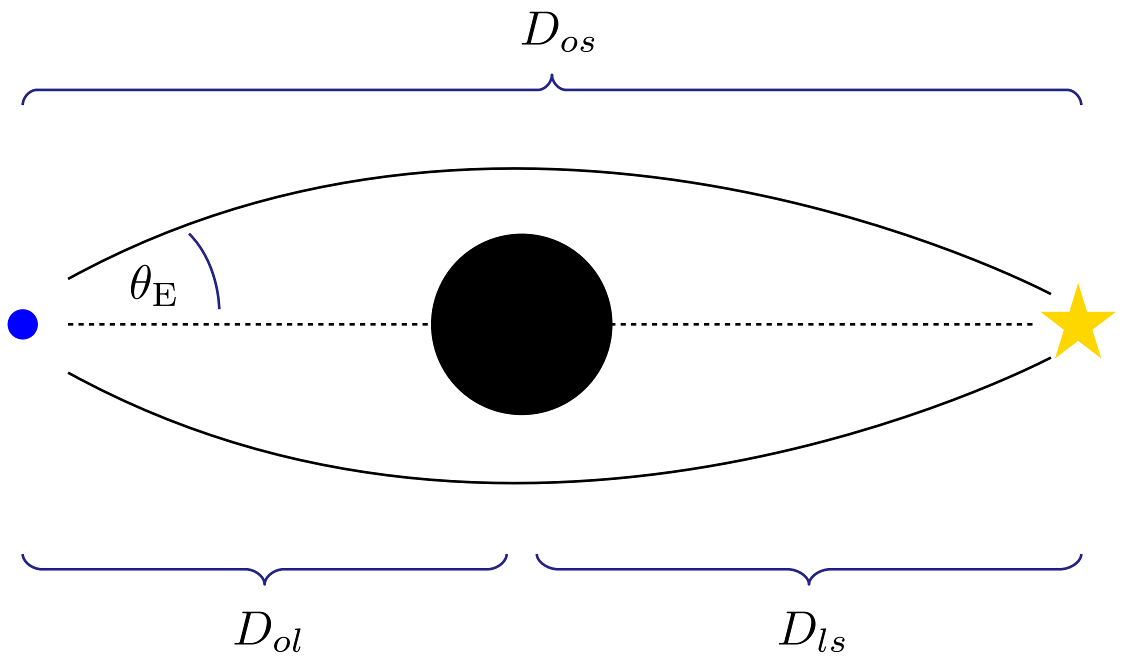
Theory of gravitational lensing (1/3)



For a pointlike lens, many aspects can be calculated analytically.

For us, it will serve as an important point of comparison for our numerical studies later.

Theory of gravitational lensing (2/3)

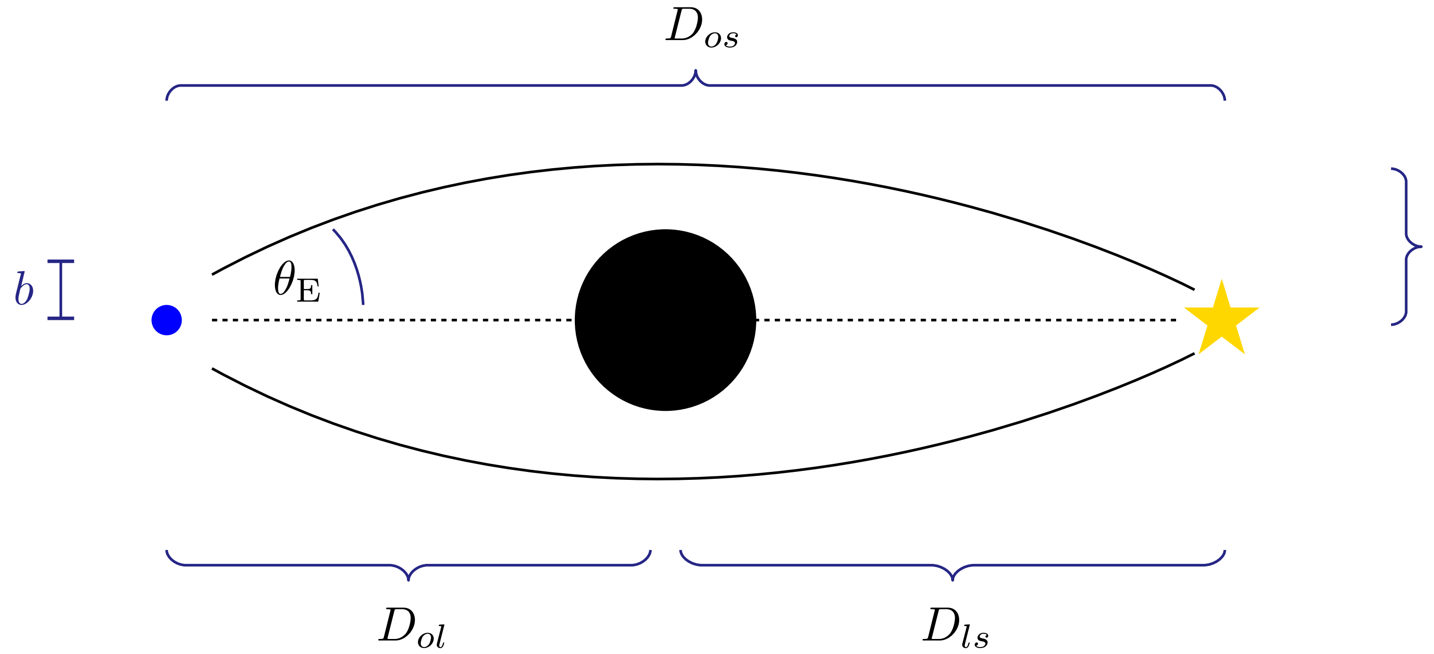


"Einstein radius" r_E

$$r_\mathrm{E} = \sqrt{\boxed{\frac{4GM}{c^2}}\frac{D_{ol}D_{ls}}{D_{os}}}$$

general relativity

Theory of gravitational lensing (2/3)

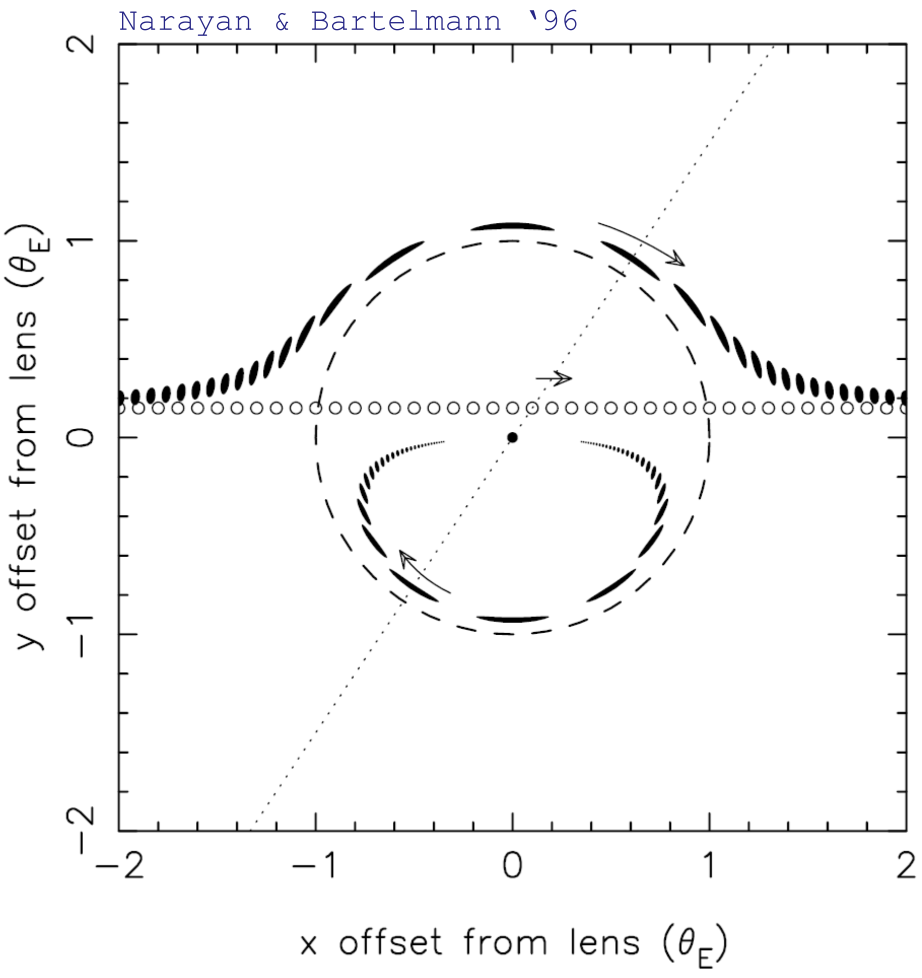


$$r_\mathrm{E} = \sqrt{\boxed{\frac{4GM}{c^2}}\,\frac{D_{ol}D_{ls}}{D_{os}}}$$

general relativity

Consider small offset **b** from observer-lens axis; define angle $\beta = \dfrac{b}{D_{ol}}$ and quantity $u = \dfrac{\beta}{\theta_\mathrm{E}} = \dfrac{b}{r_\mathrm{E}}$.
Deviation from axis creates double images.

Theory of gravitational lensing (3/3)



Angular locations of double images:

$$\theta_\pm = \frac{1}{2}\left[\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right]$$

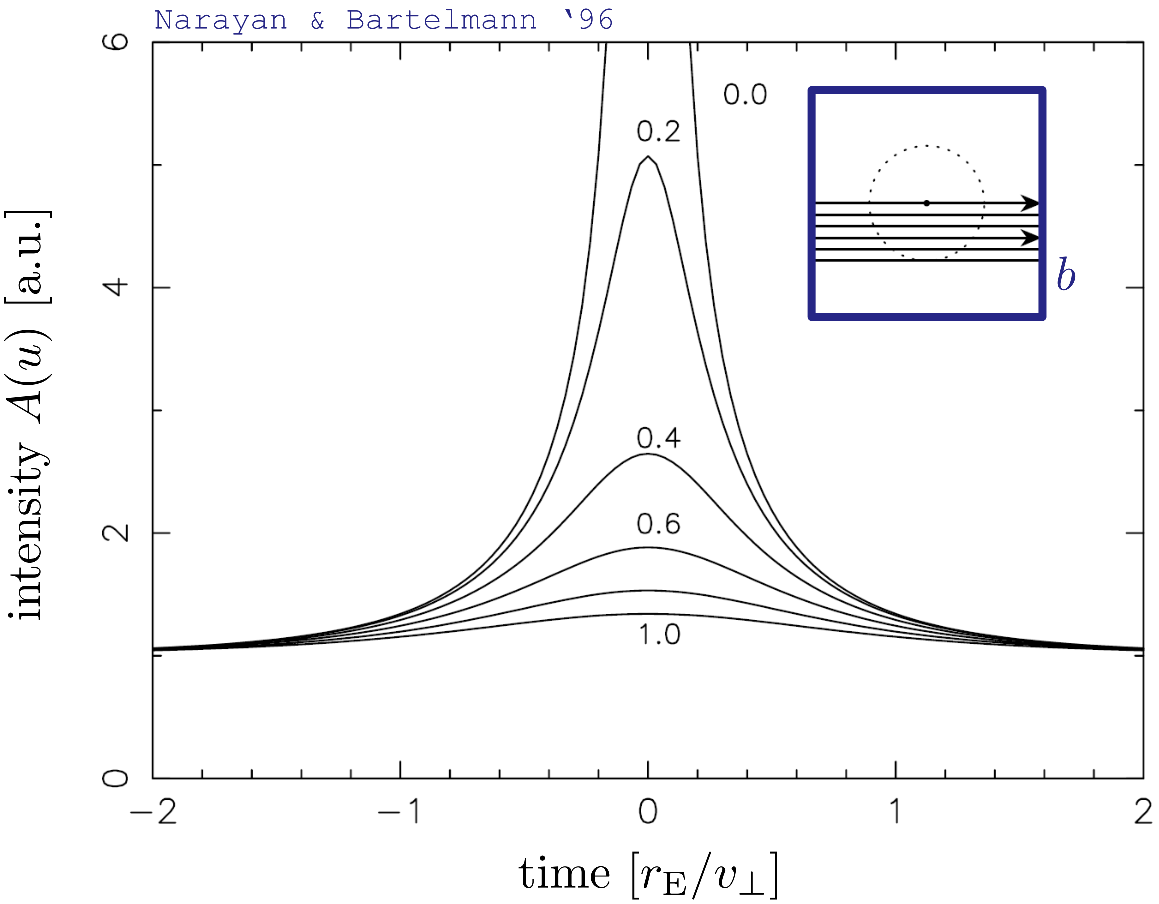
Total amplification is a sum of two images:

$$A = A_+ + A_- = \frac{u^2+2}{u\sqrt{u^2+4}},$$

$$u = \frac{\beta}{\theta_E} = \frac{b}{r_E}.$$

This is sufficient to compute the intensity, which is proportional to the amplification.

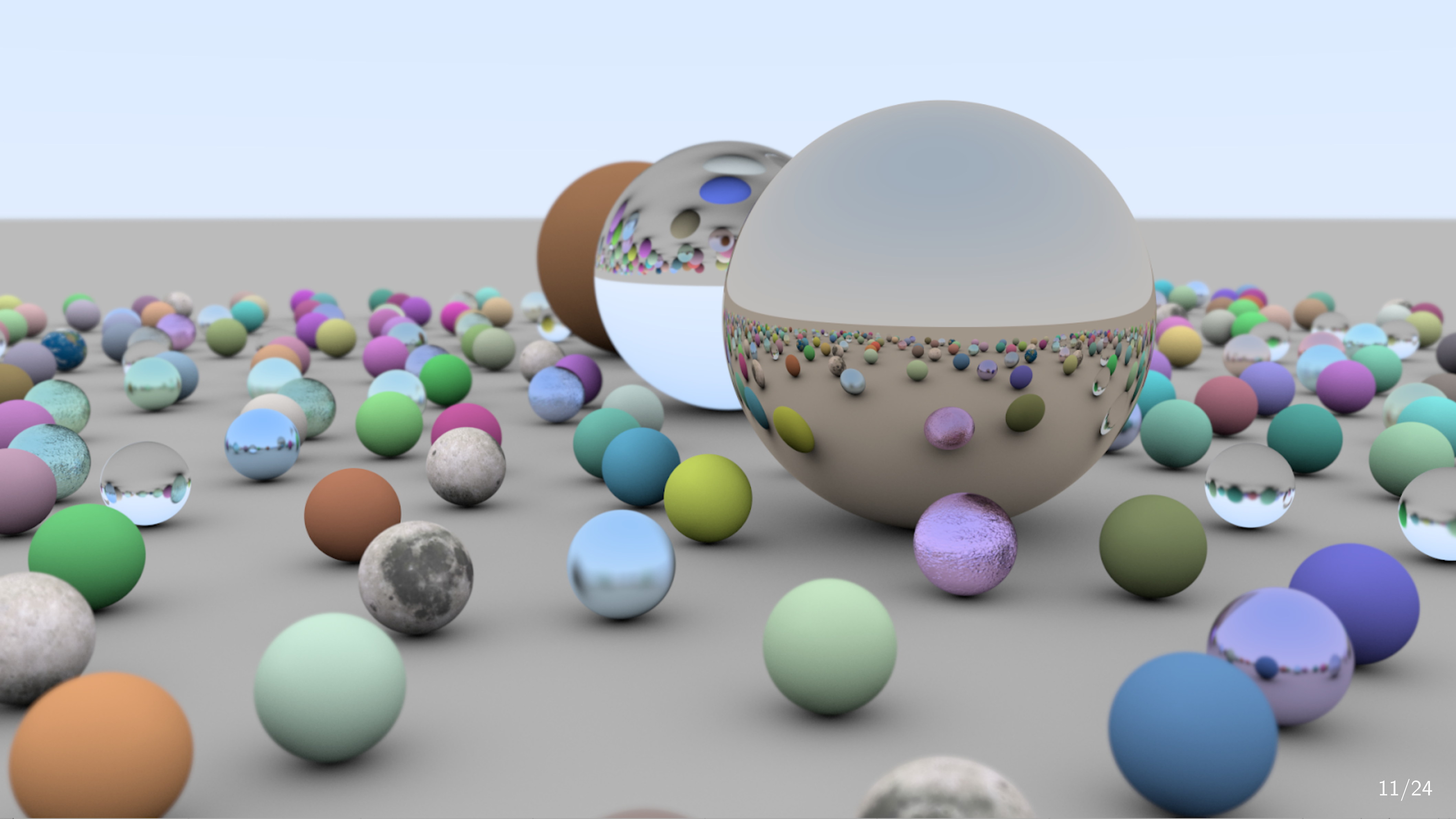
The light curve: "smoking gun" of gravitational microlensing



$$A(u) = \frac{u^2+2}{u\sqrt{u^2+4}}\,,$$

$$u = \frac{\beta}{\theta_{\rm E}} = \frac{b}{r_{\rm E}}\,.$$

3/5 Ray tracing




softsys
HOテル
天下無双
BODY WITHOUT LIMITS
RTX
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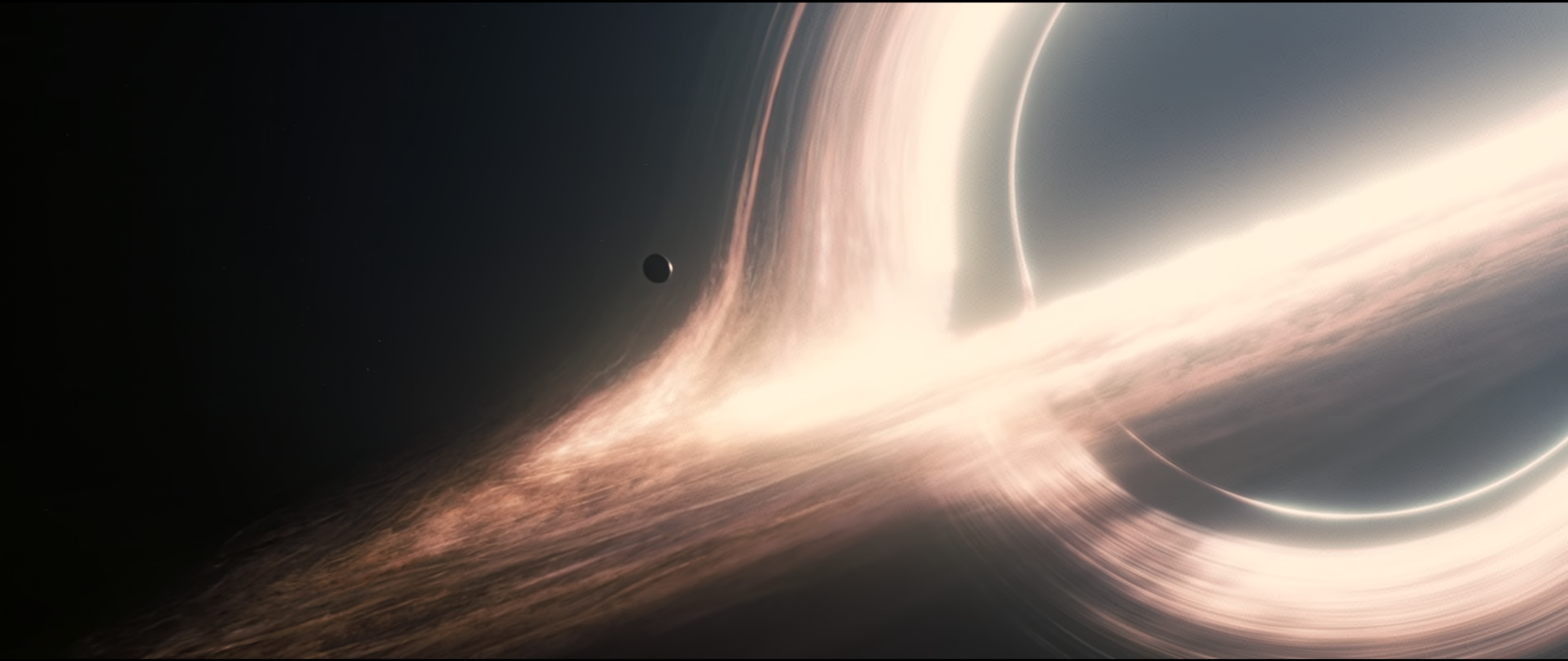

Ray tracing in a nutshell

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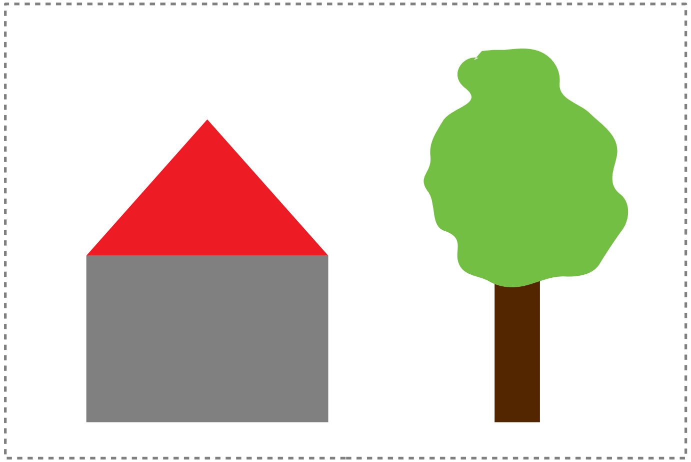
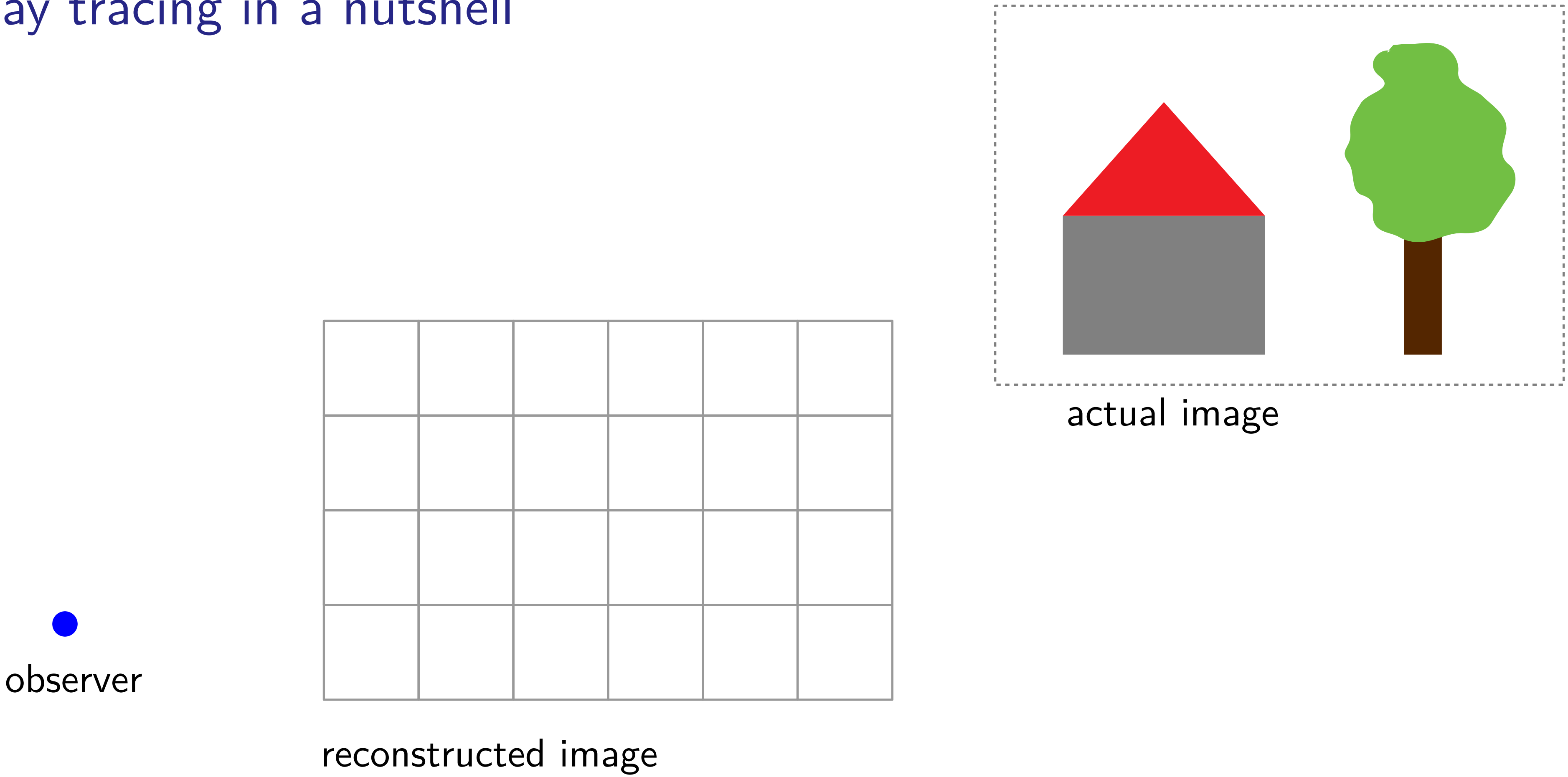


actual image

observer

Ray tracing in a nutshell



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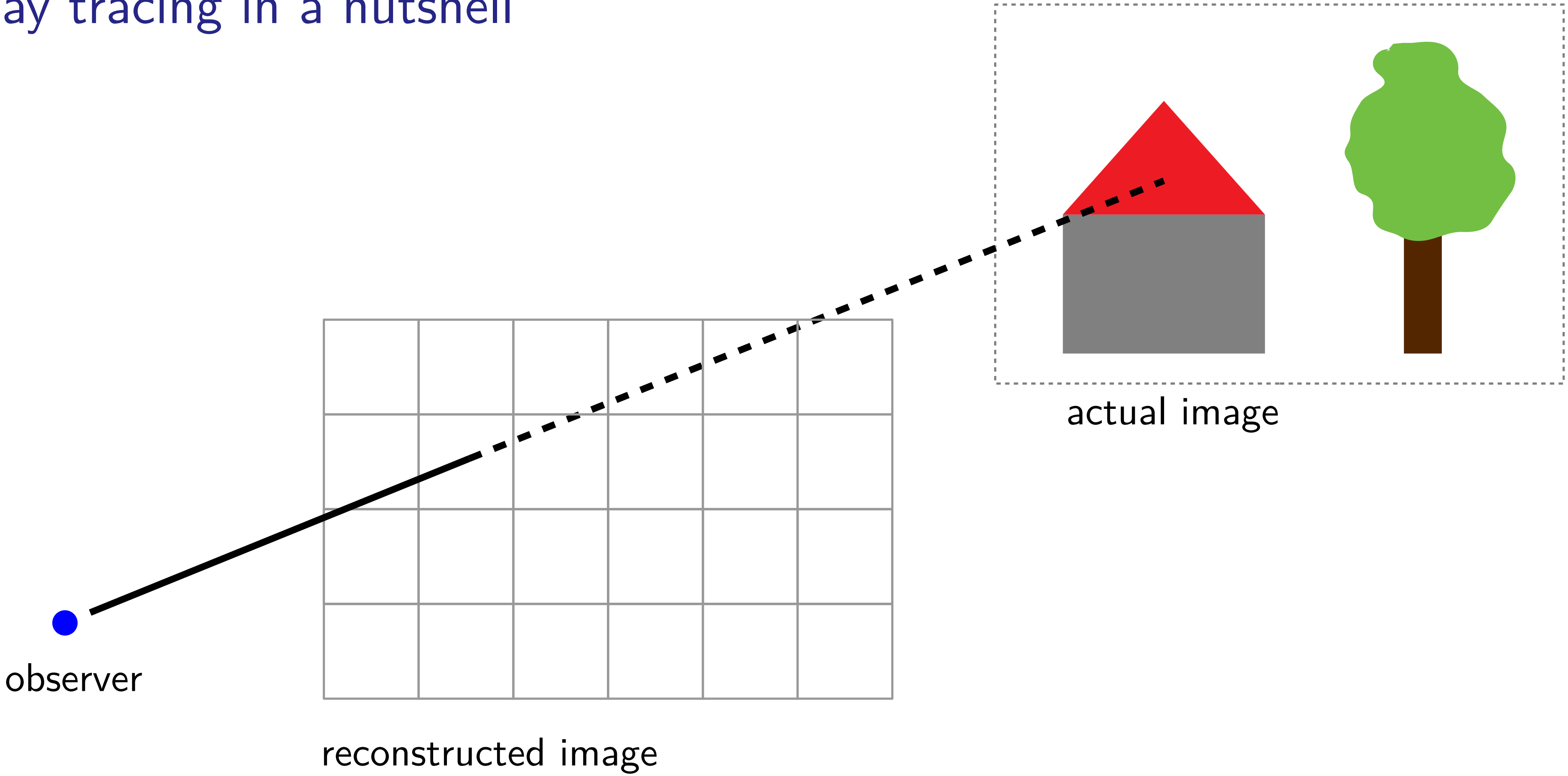
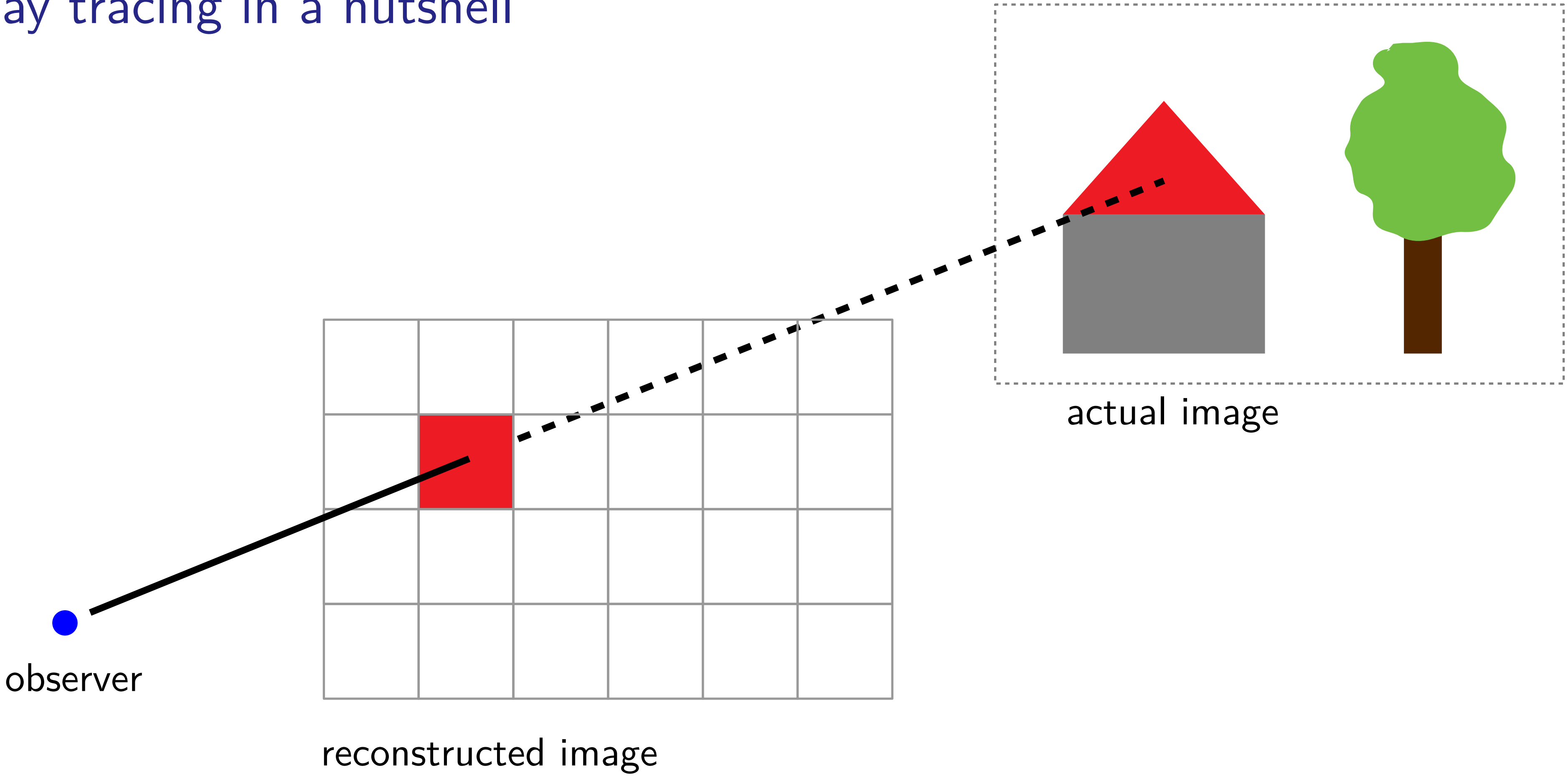
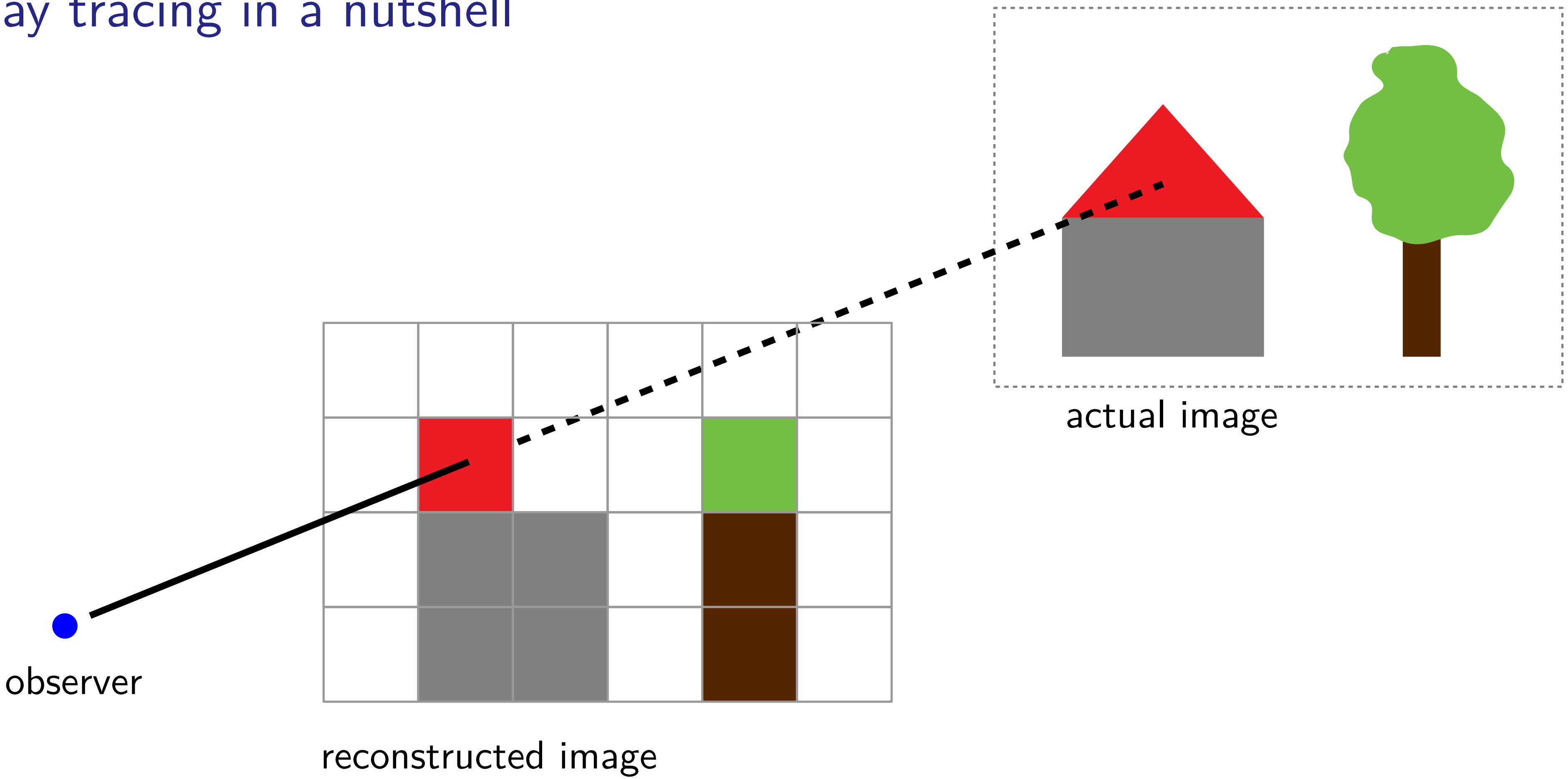
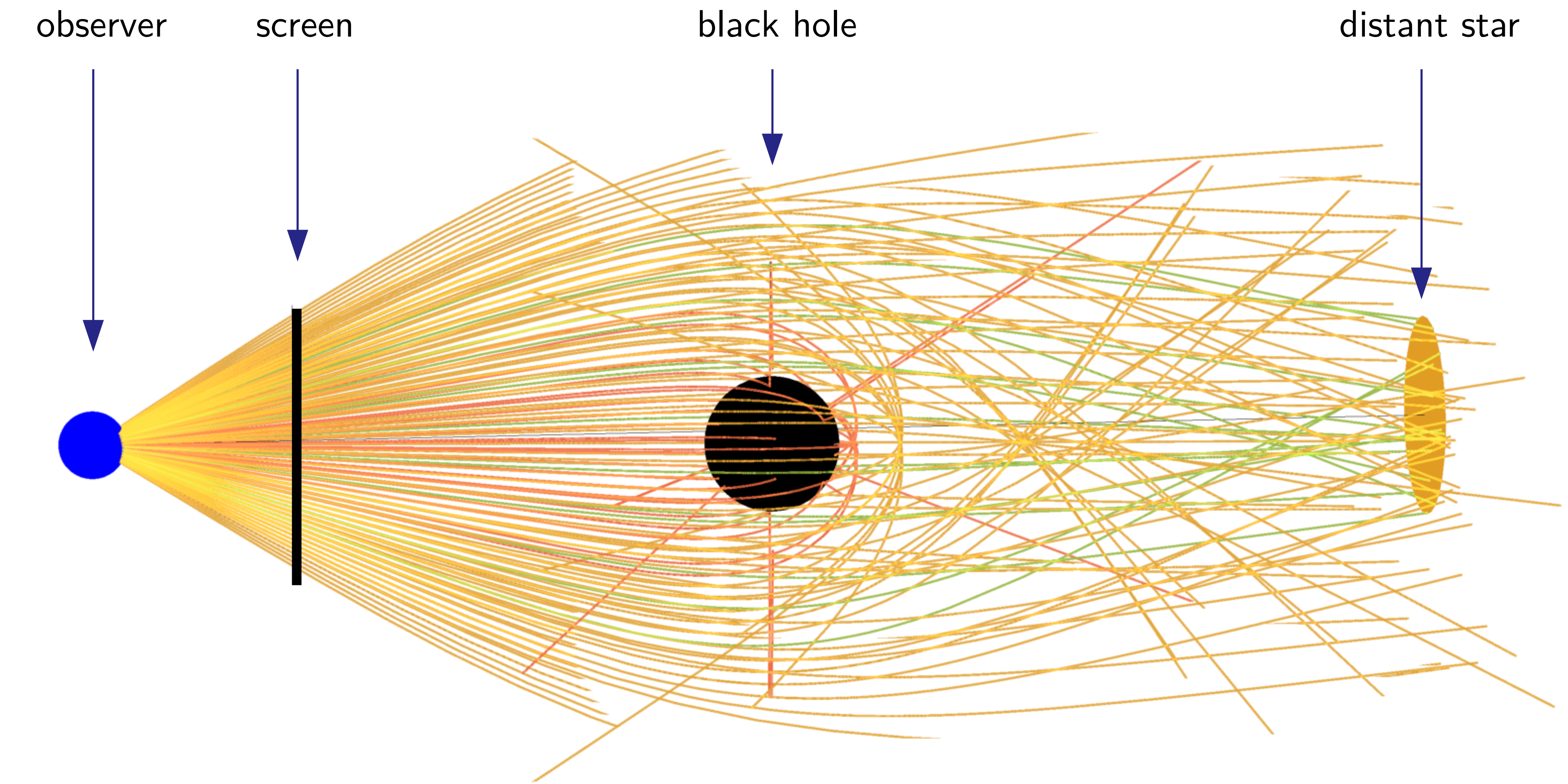


Ray tracing in a nutshell

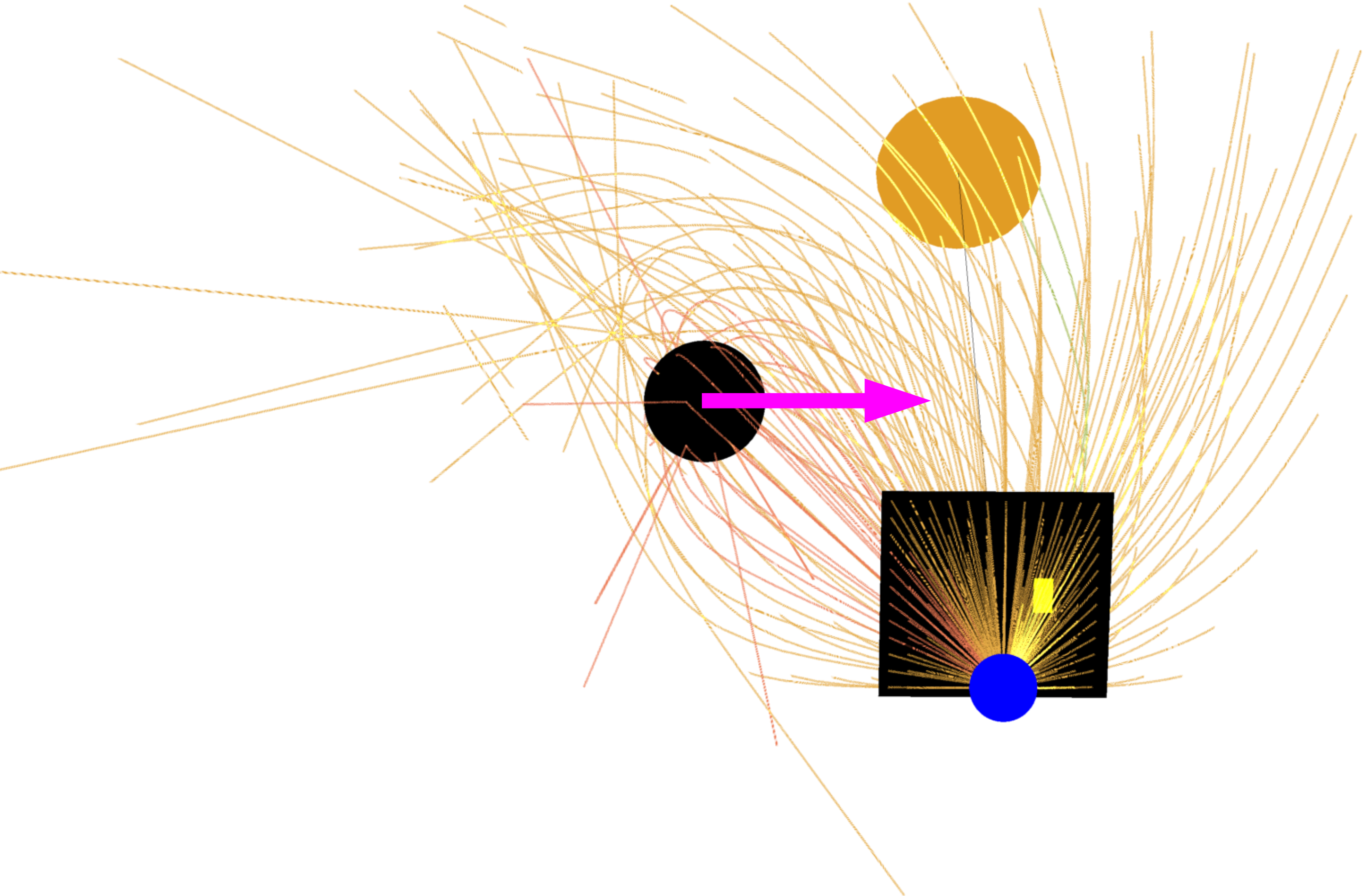


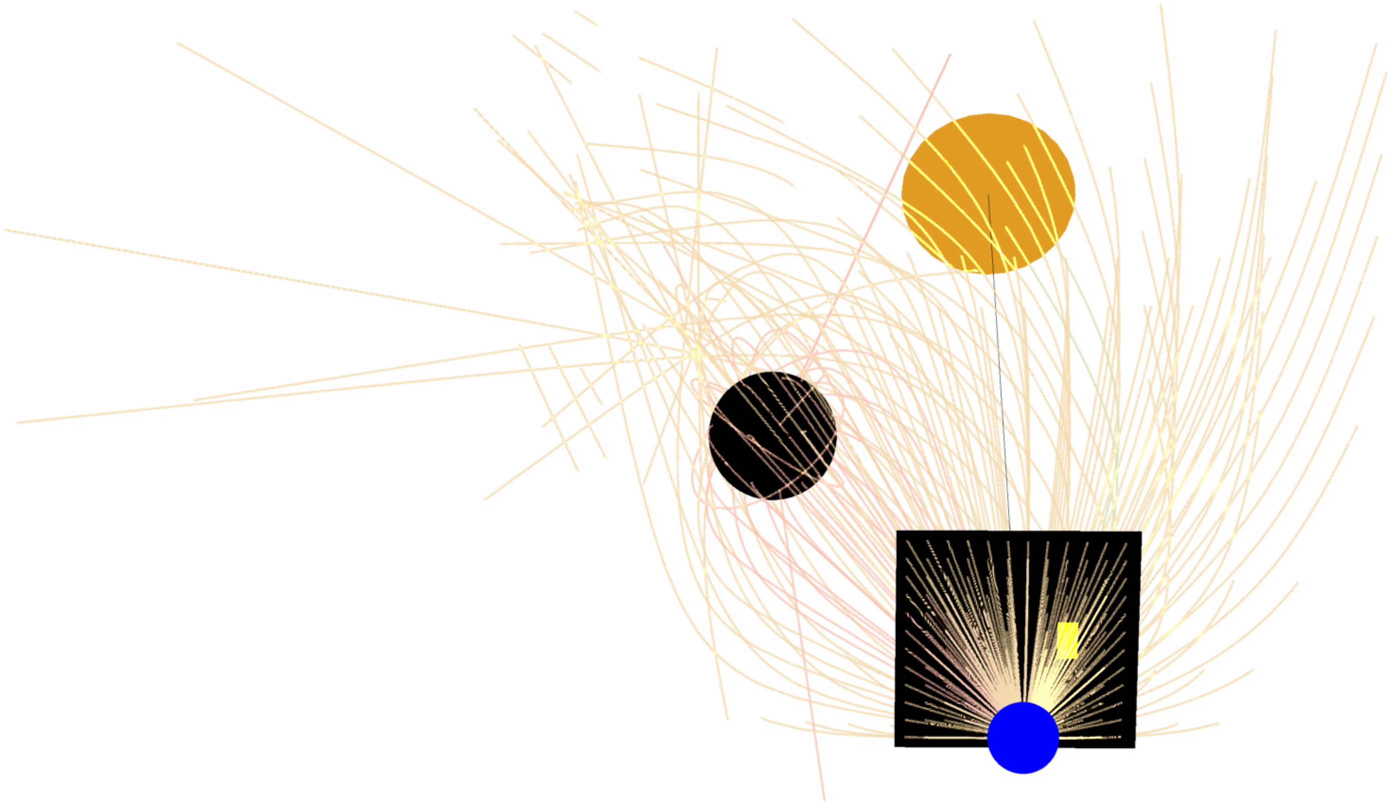
Ray tracing in a nutshell

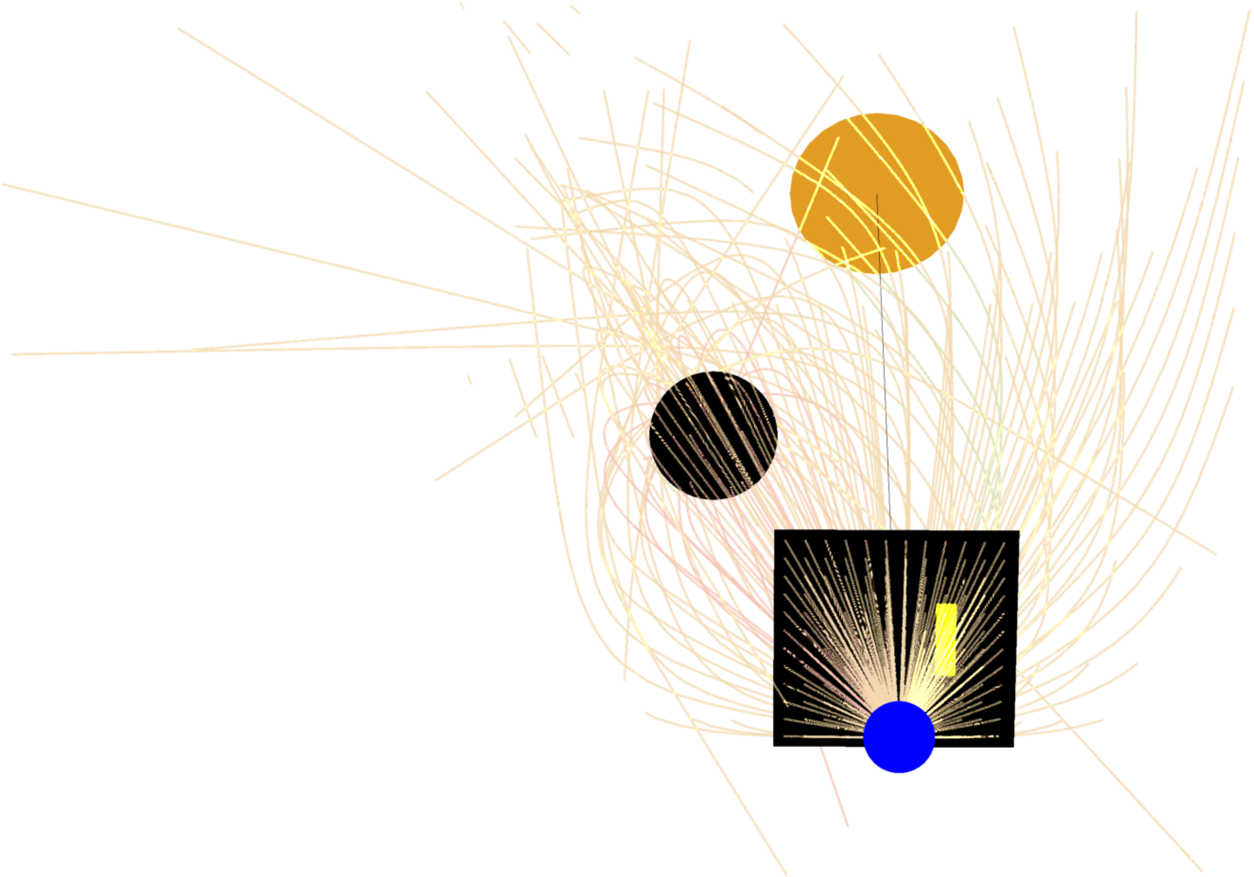


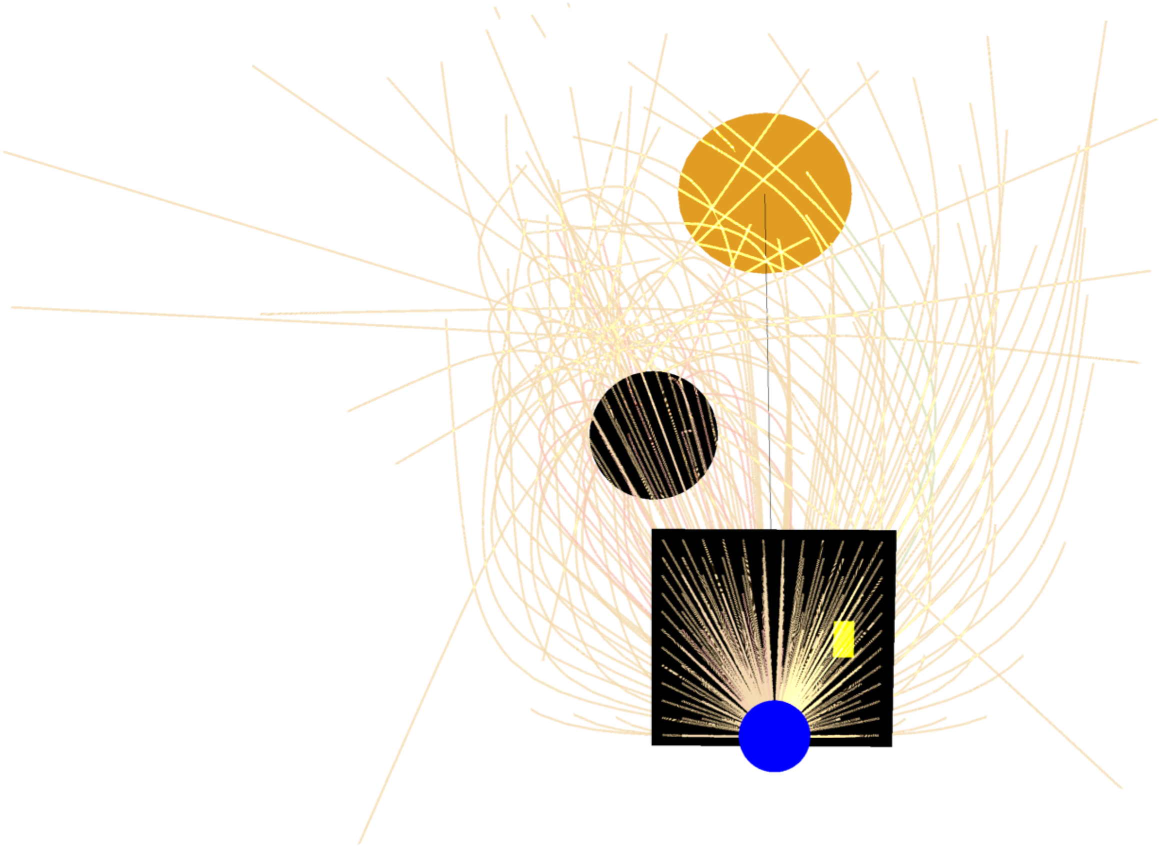


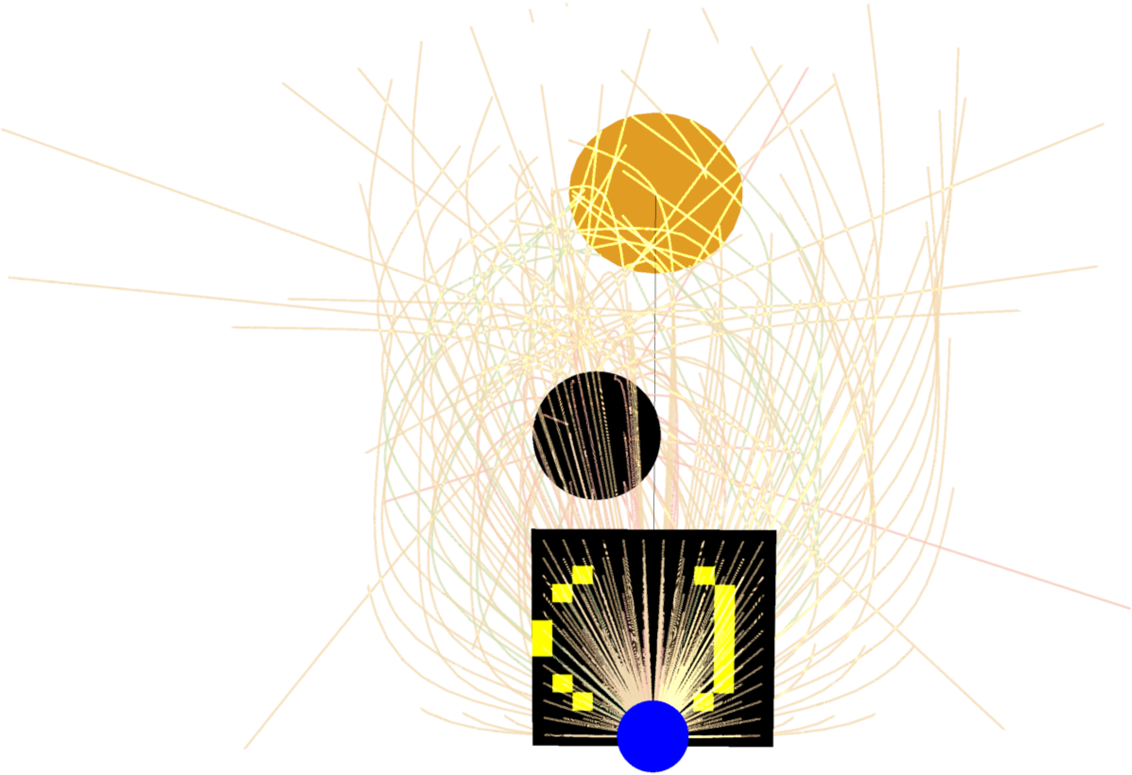
The light rays **hit the source** or **escape to infinity** or **fall into the black hole**.

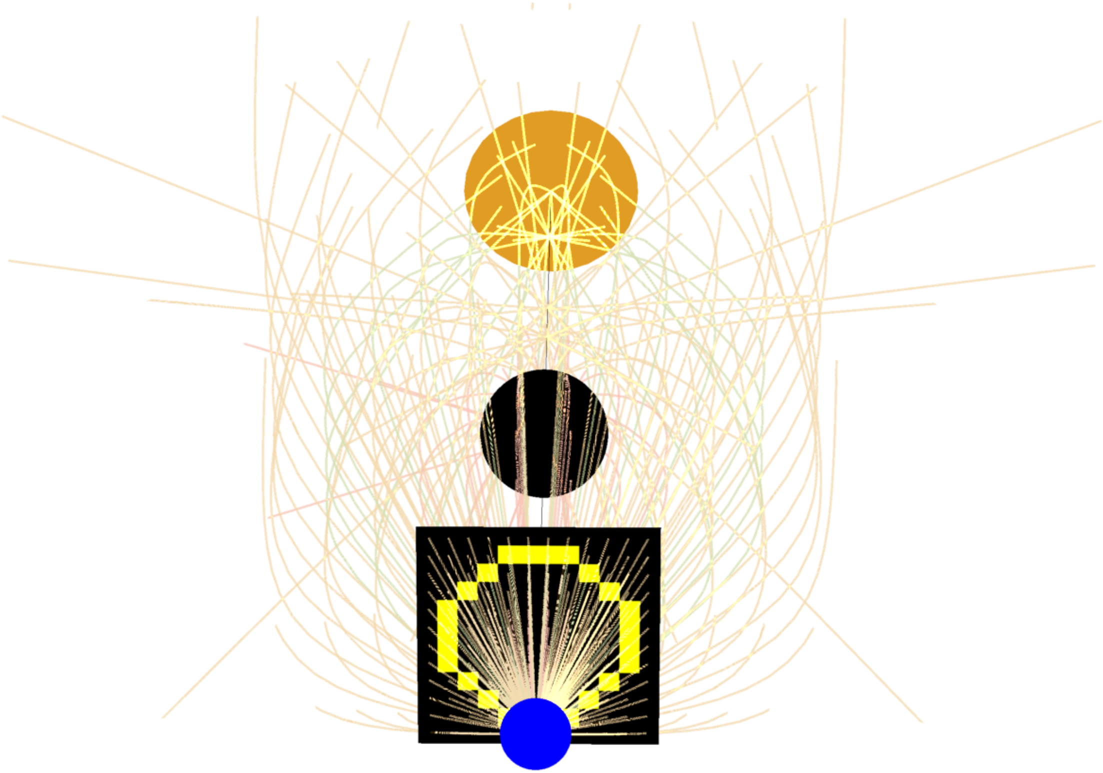


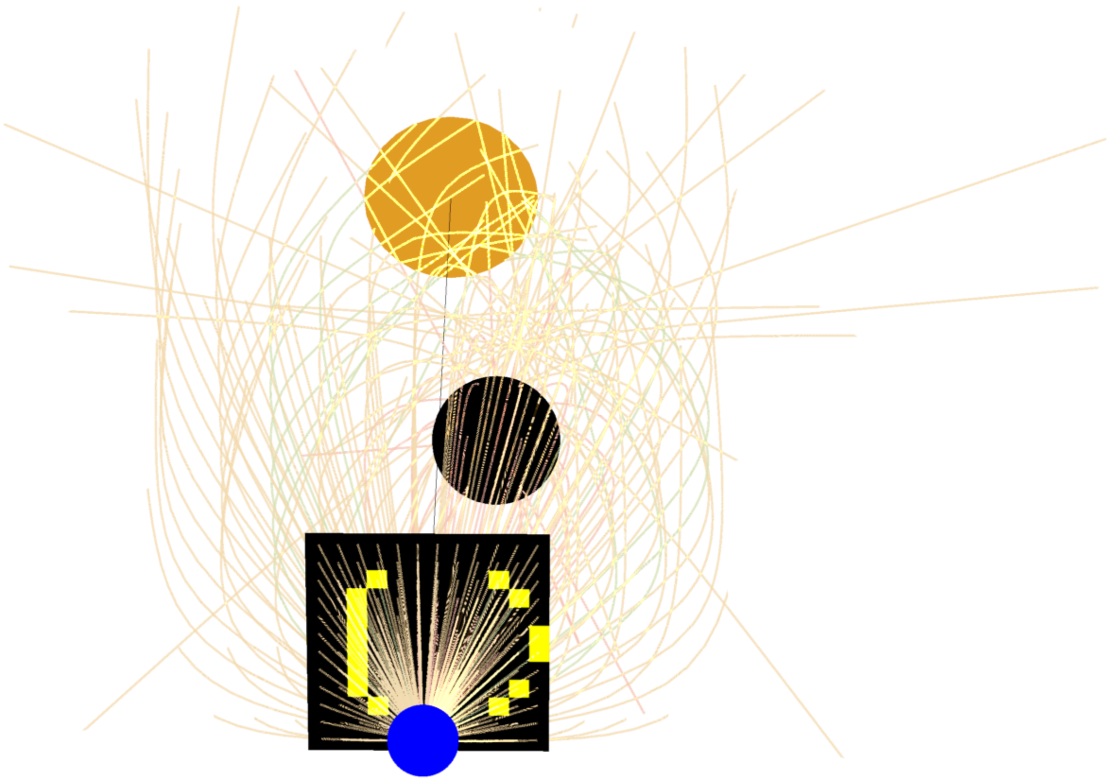


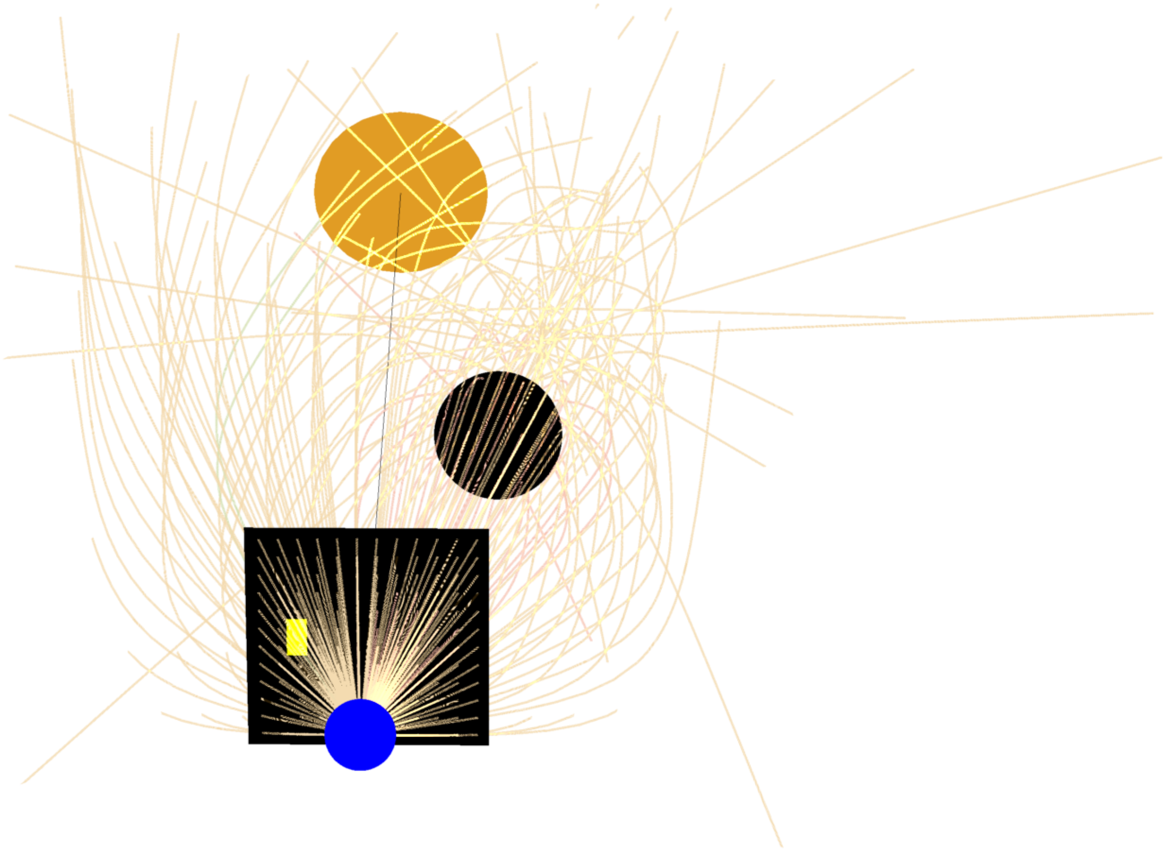


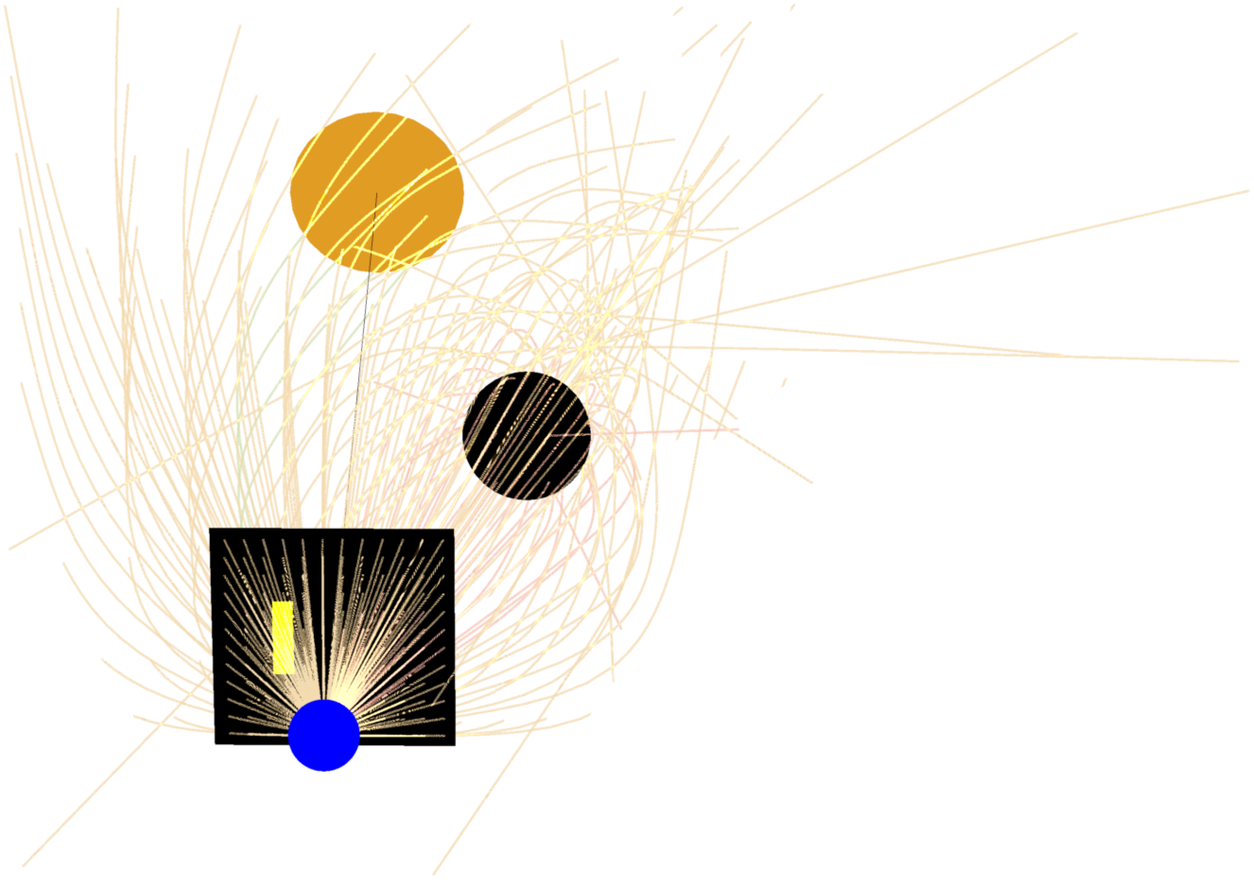


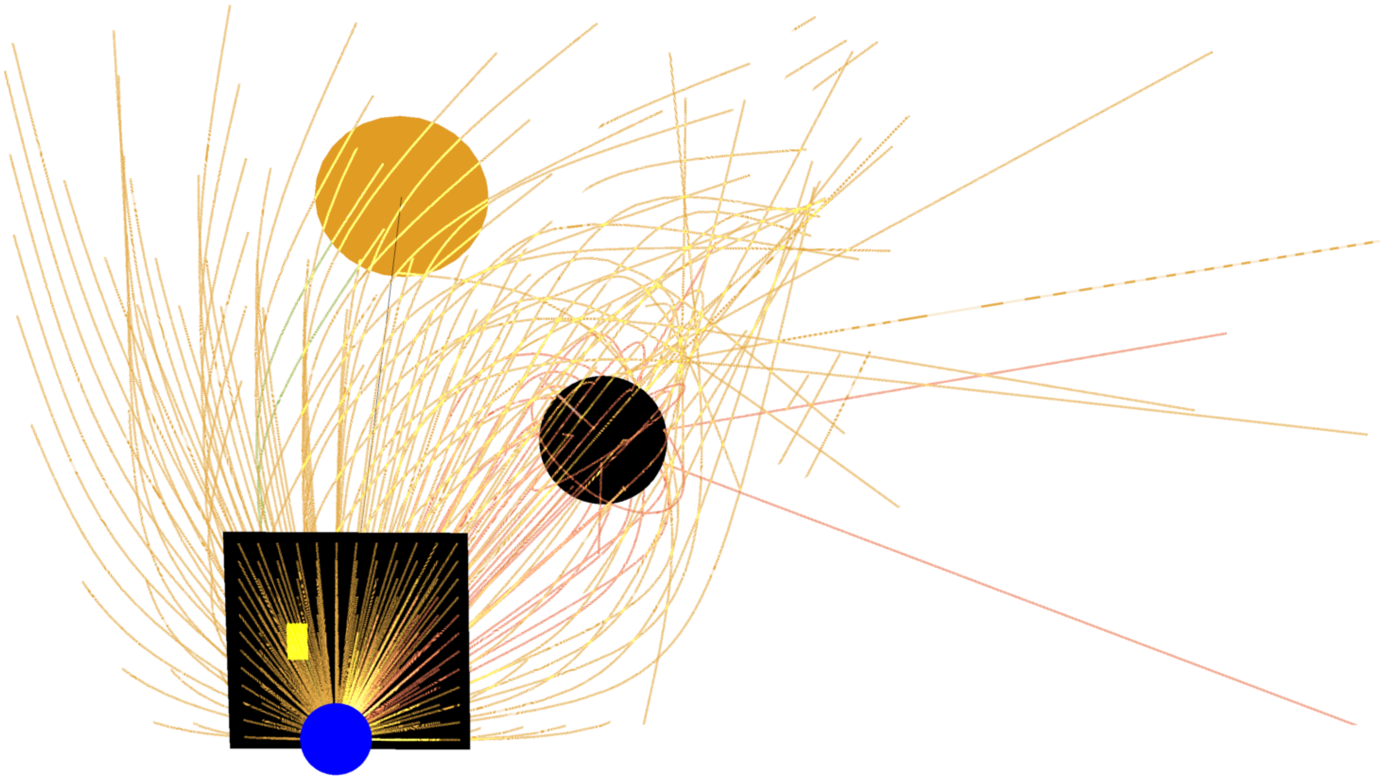


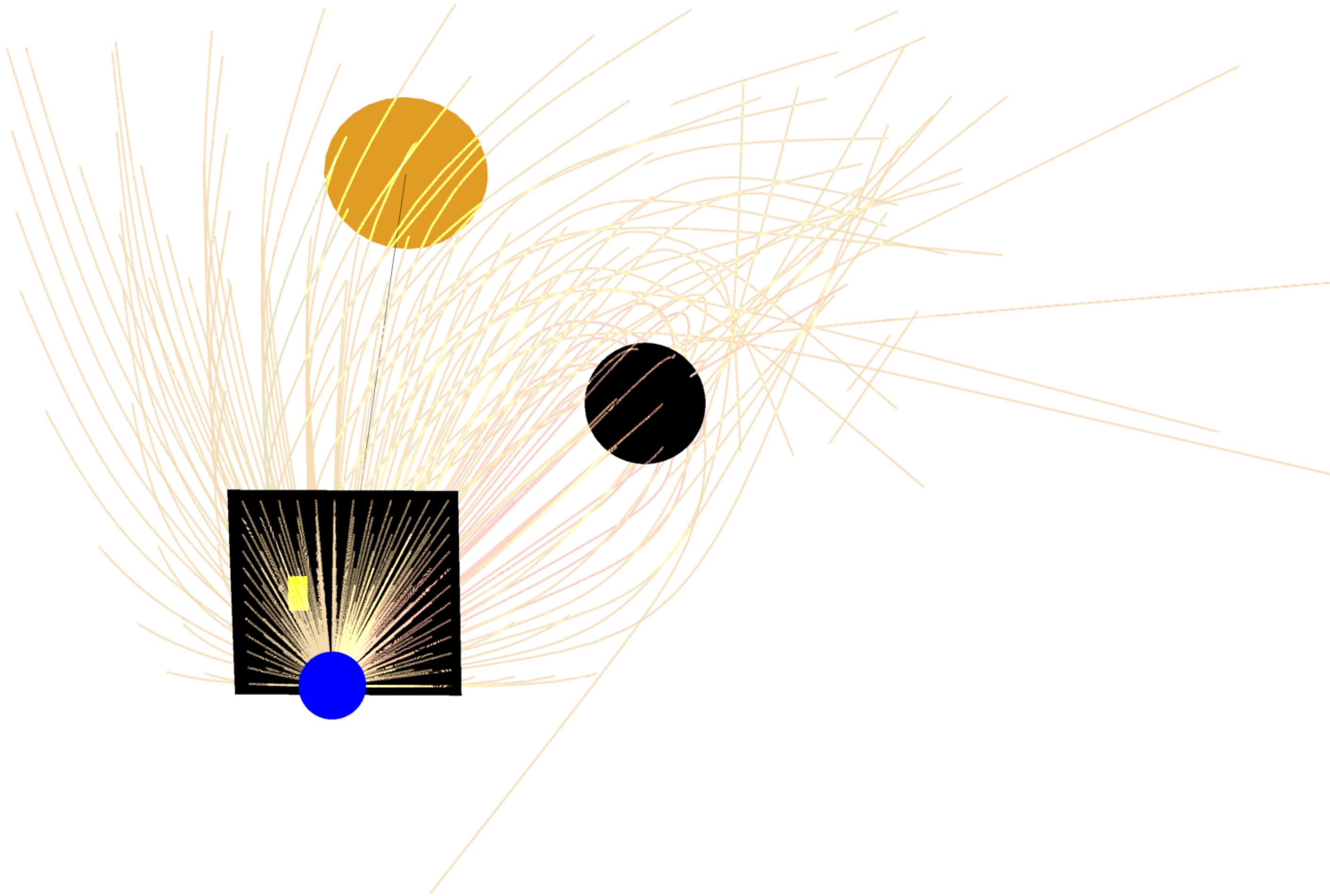




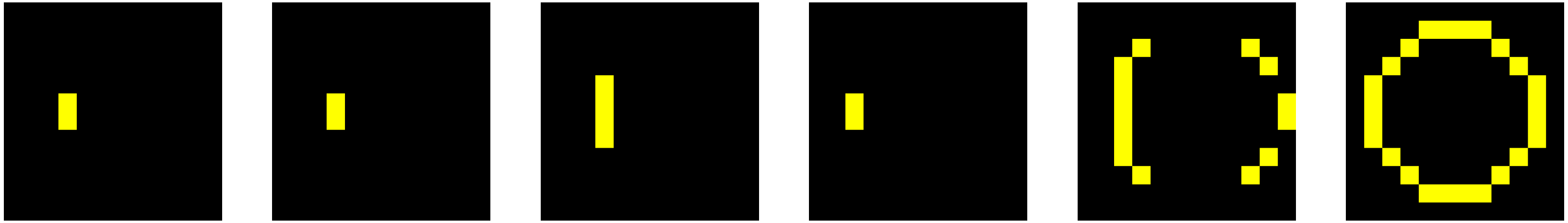








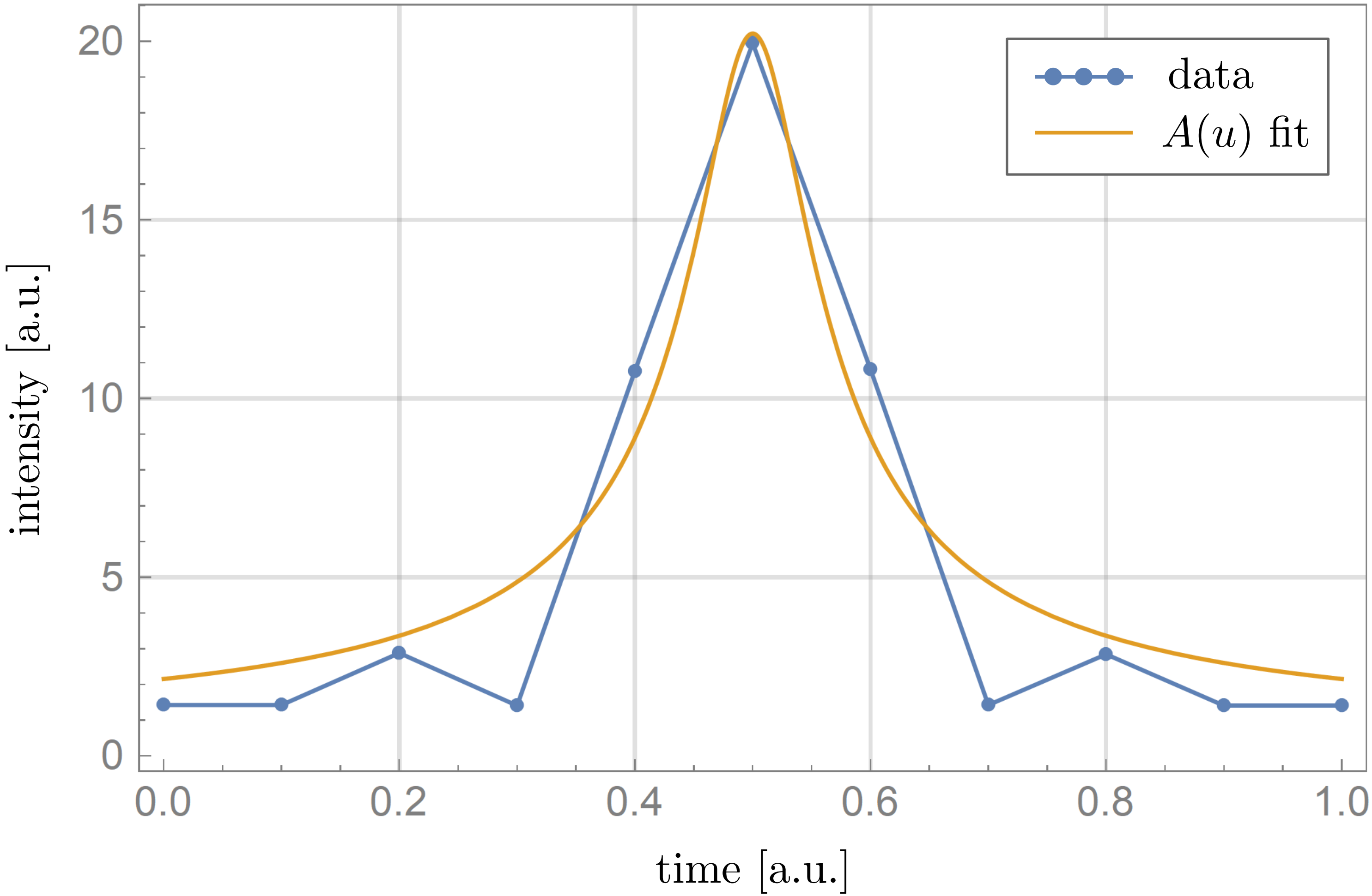
Extracting the light curve (1/2)



$T = 0.0$ $T = 0.1$ $T = 0.2$ $T = 0.3$ $T = 0.4$ $T = 0.5$

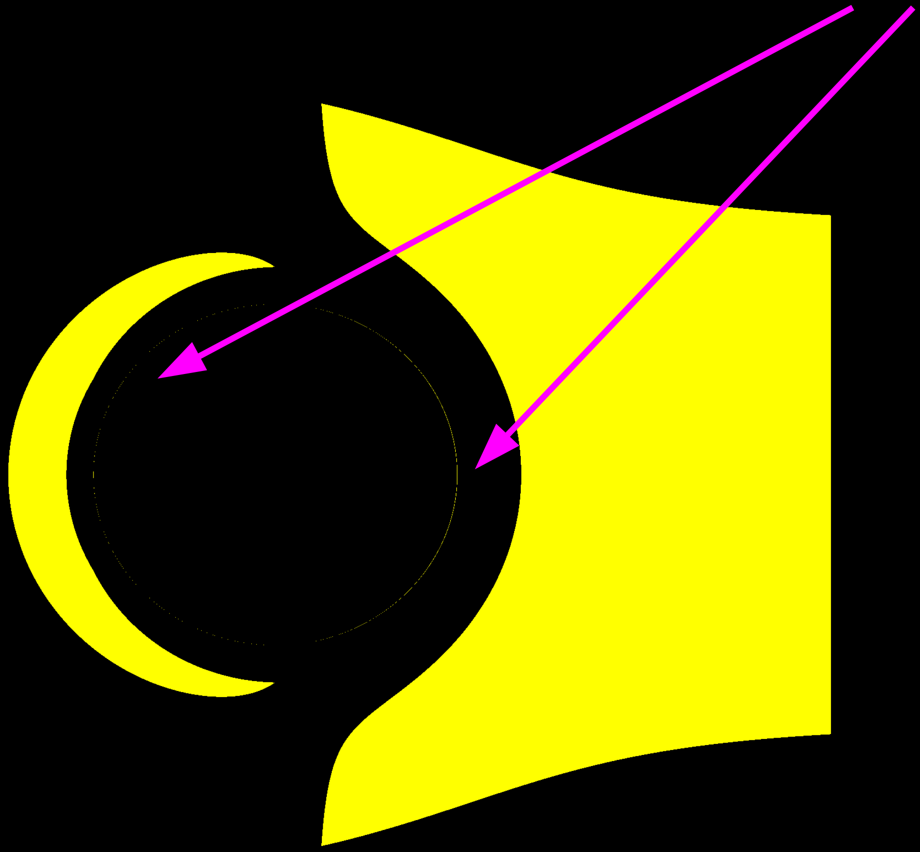
$$\text{Intensity} \propto \frac{\#\text{ of yellow pixels}}{\#\text{ of all pixels}}$$

Extracting the light curve (2/2)

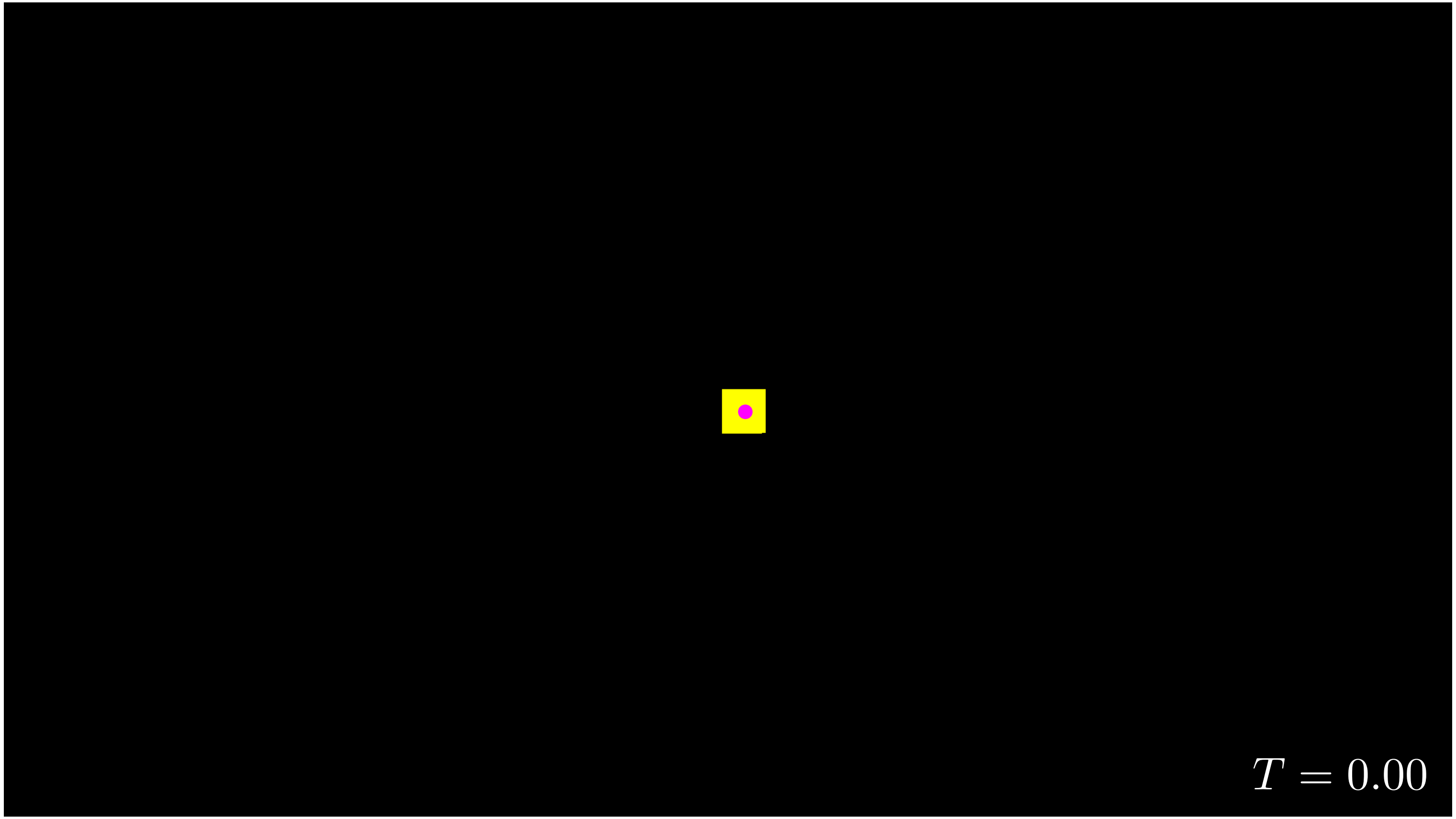


4/5 Actual results

Result #1: High-resolution renders give correct Einstein ring



Result #2: Verification of numerical setup via double images



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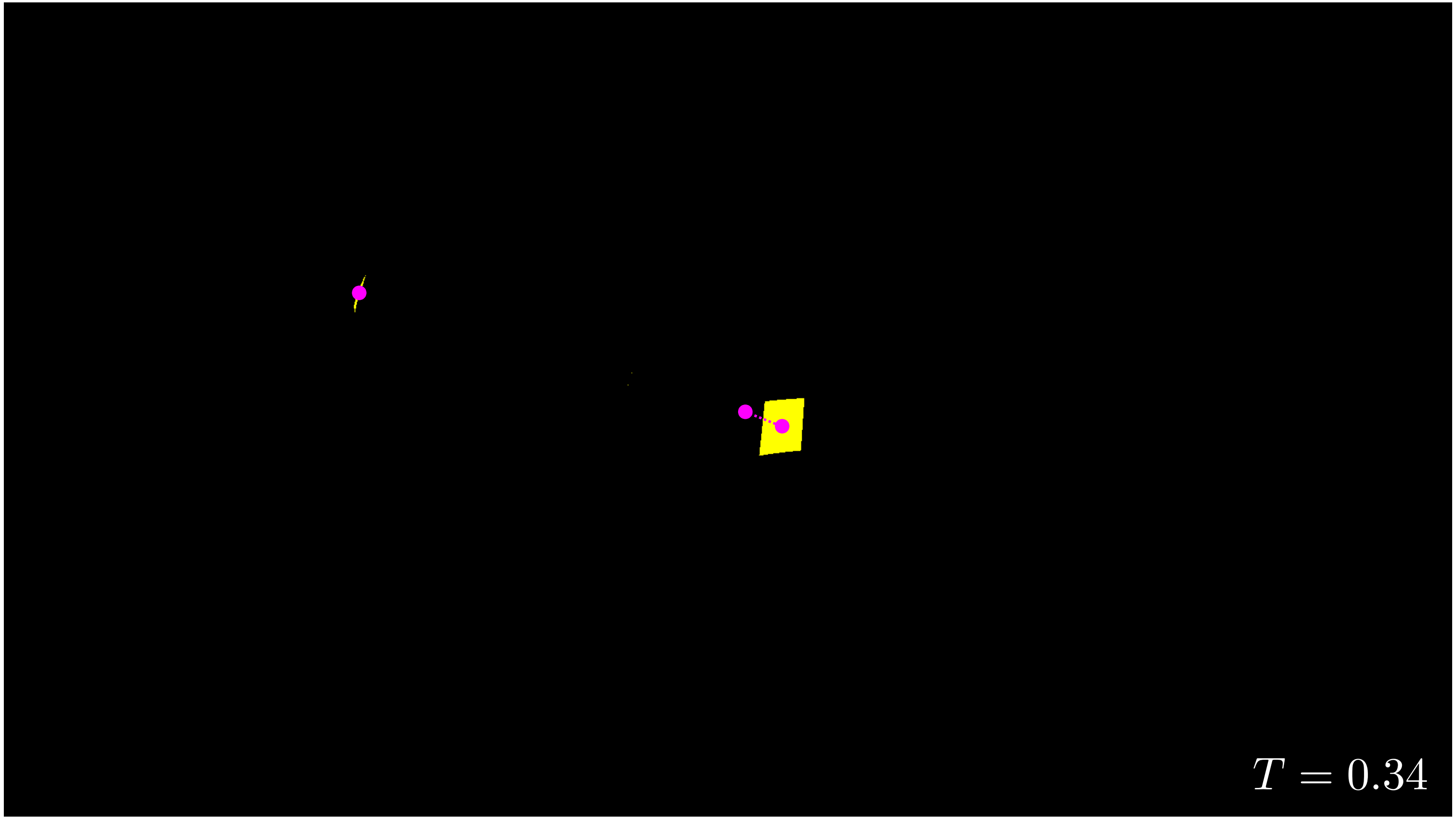

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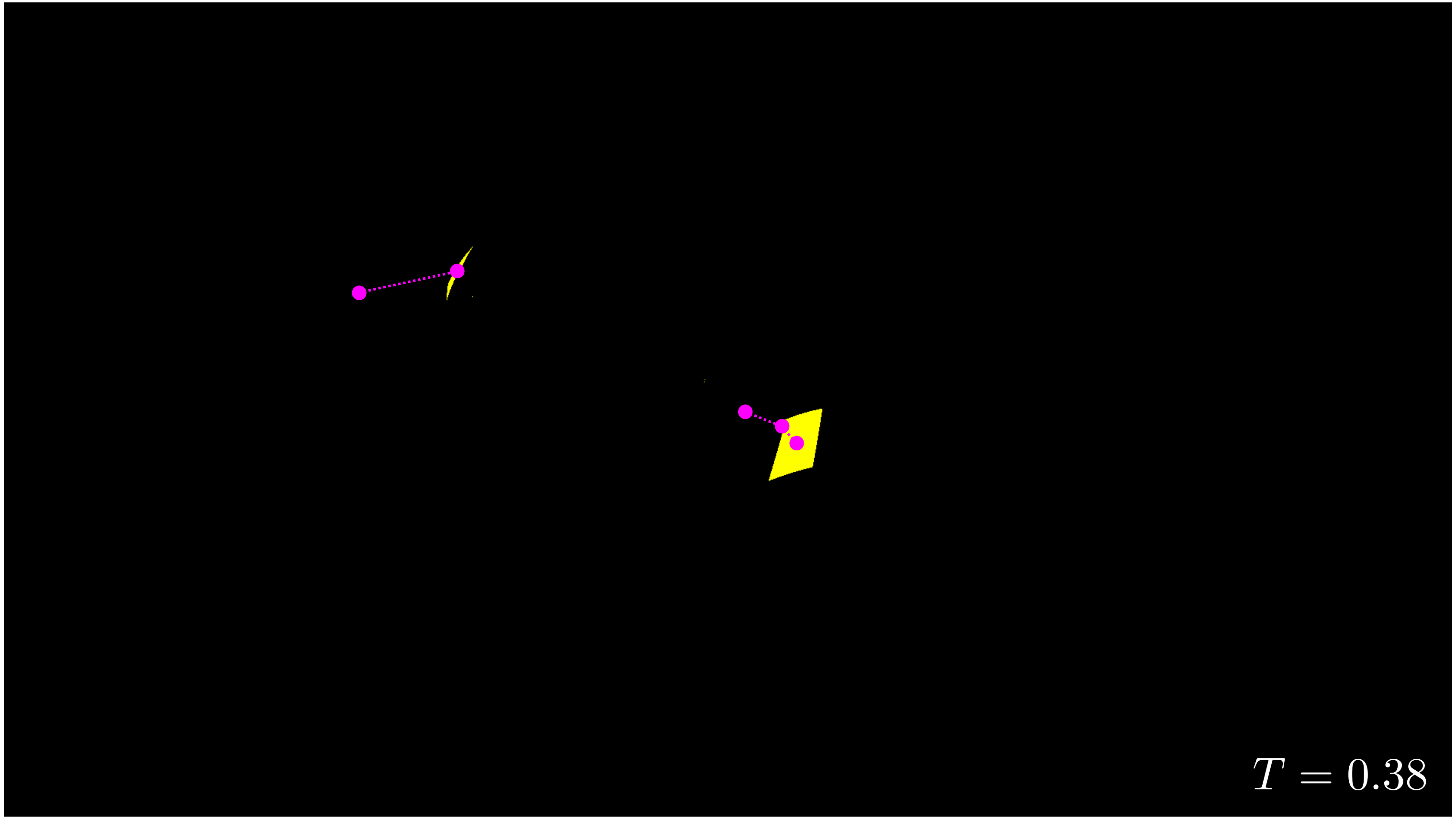

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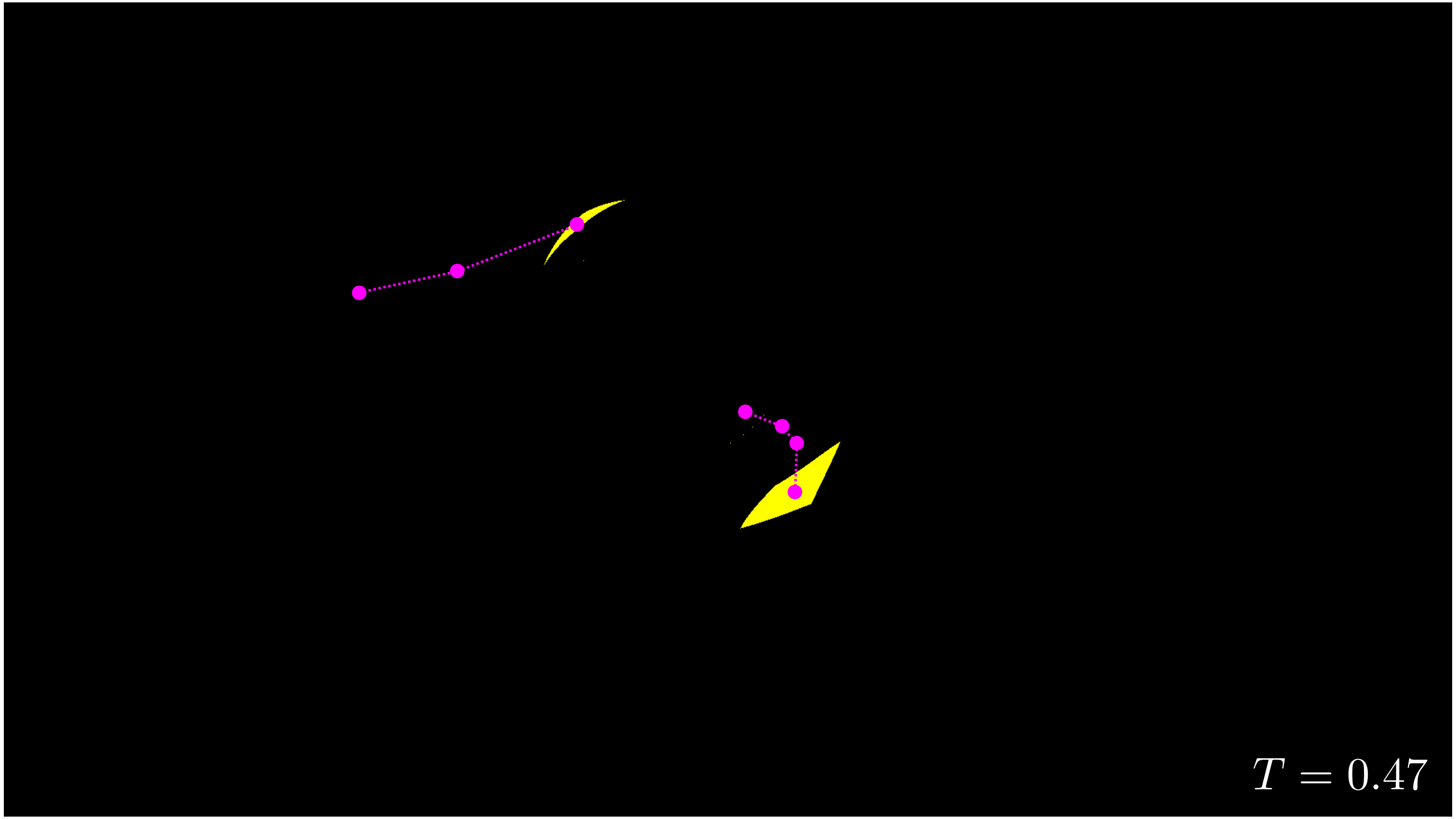

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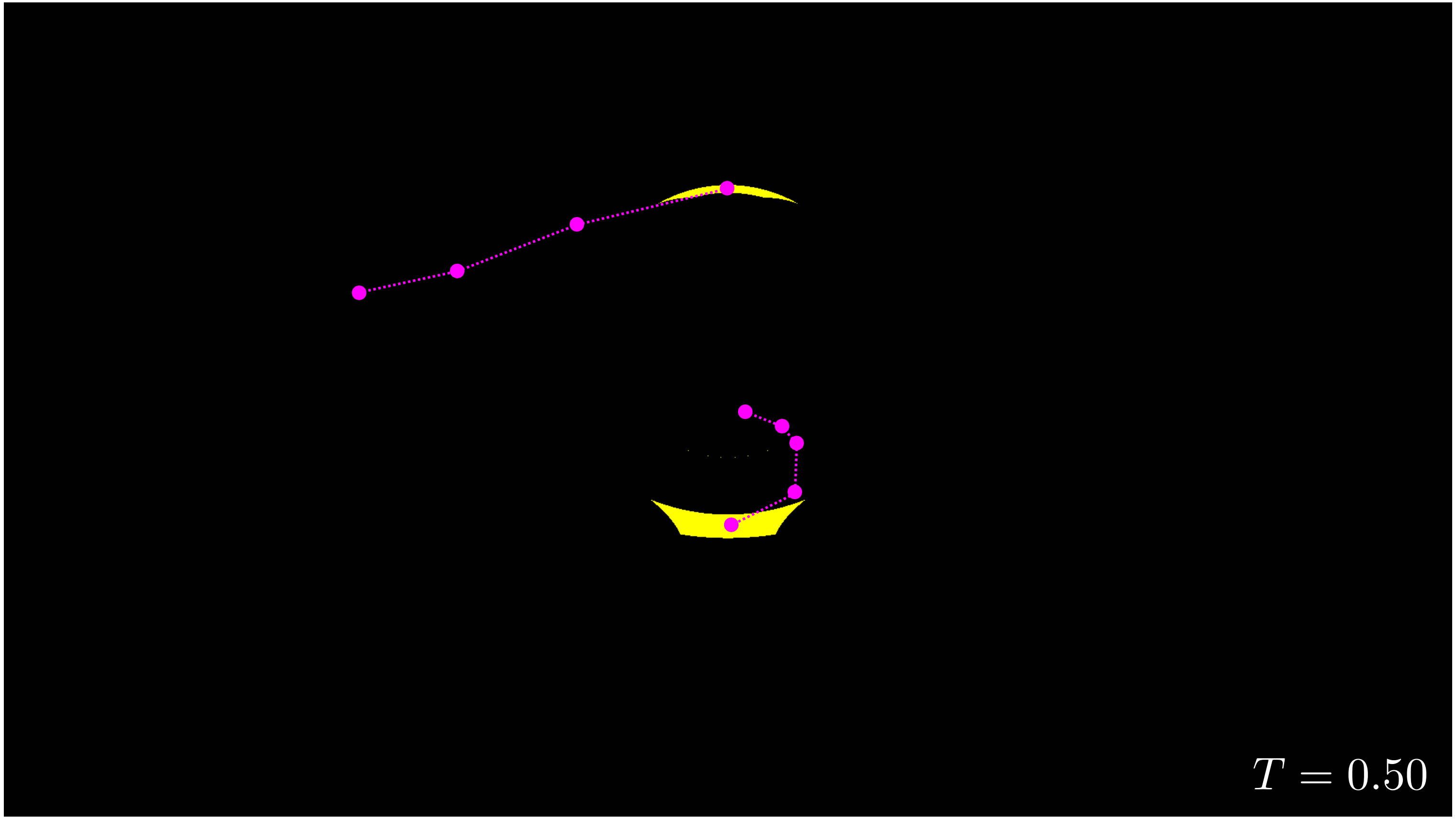

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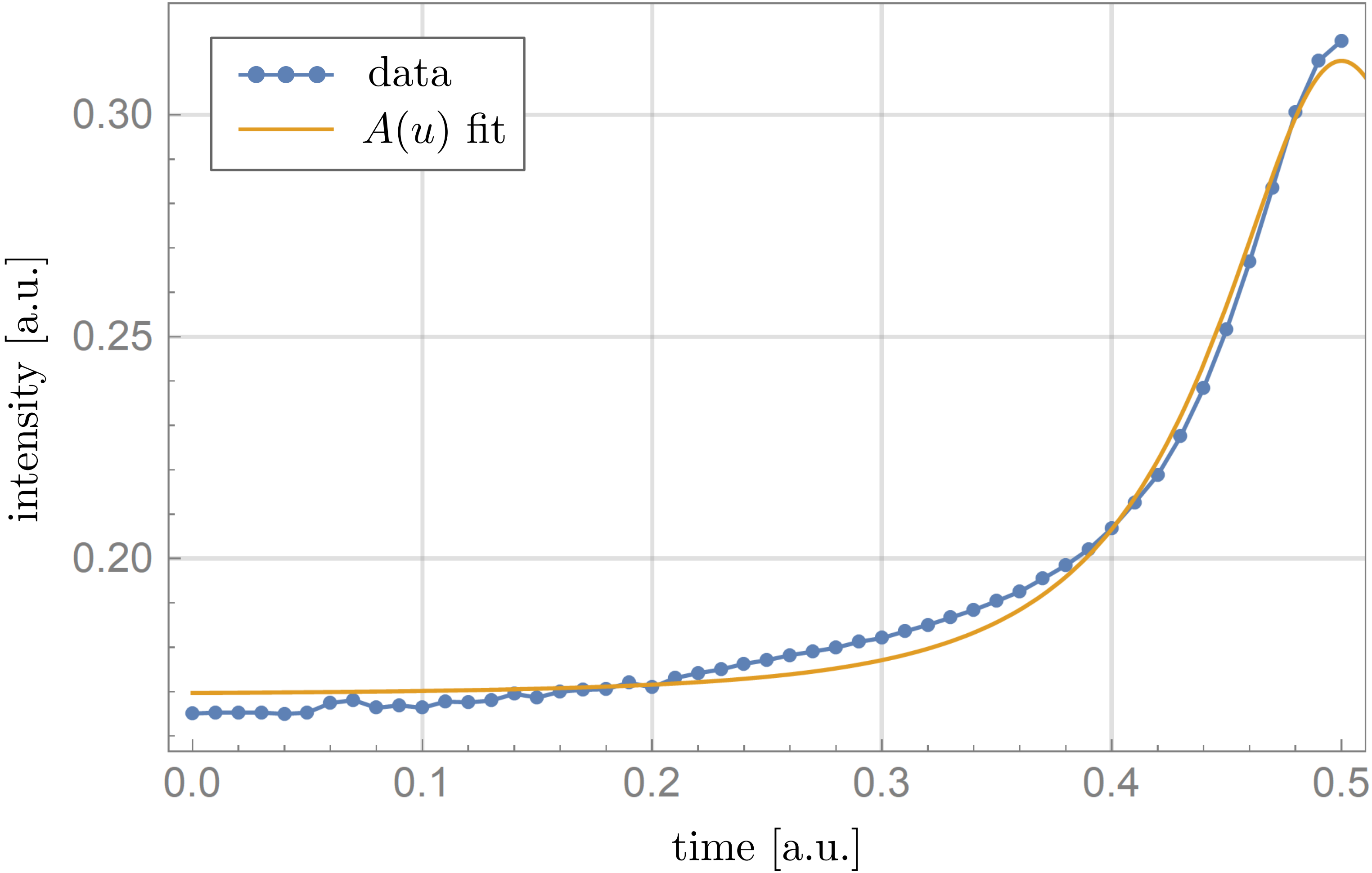
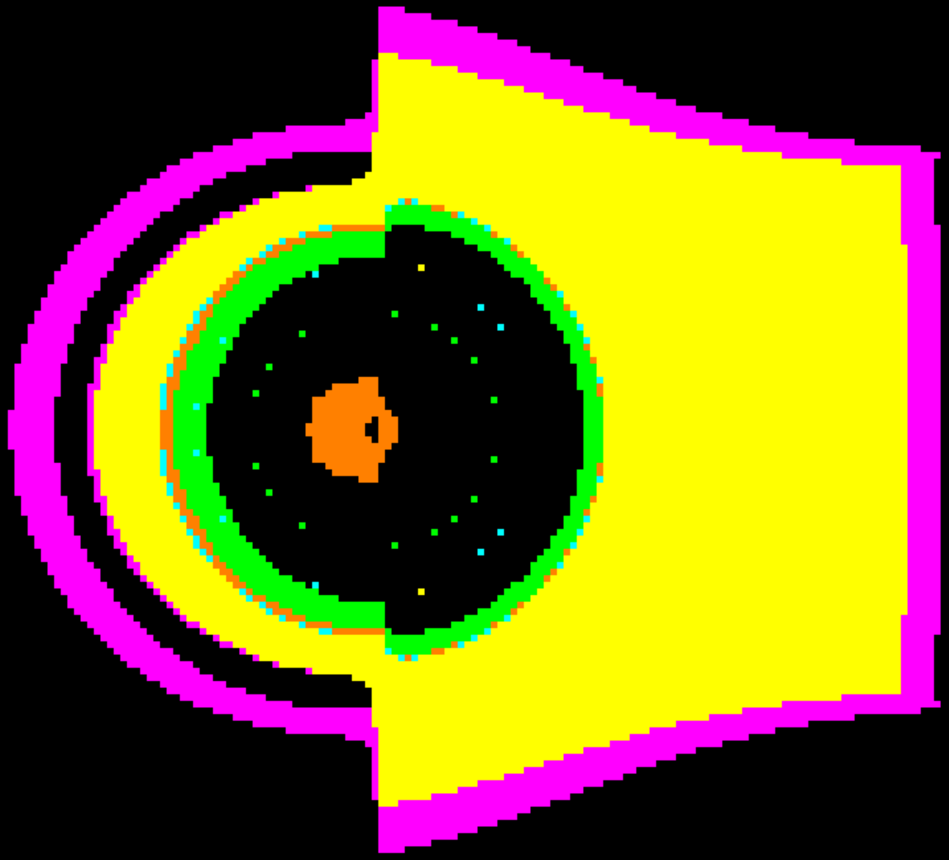


Result #3: Testing different non-singular black hole models

Resolution: 320x180



Models: Schwarzschild, bump metric, Hayward, horizonless Hayward, Minkowski core.

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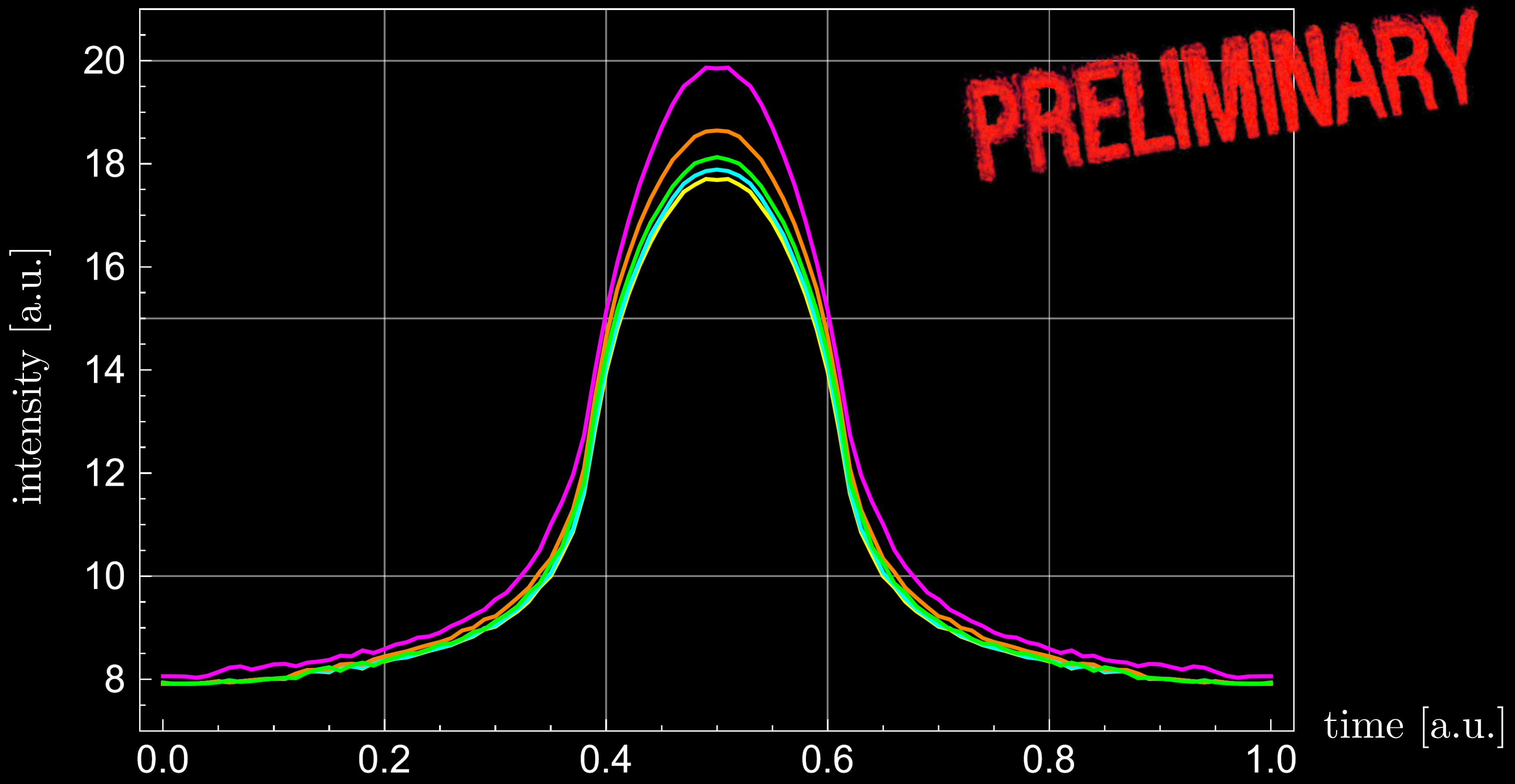


Models: Schwarzschild, bump metric, Hayward, horizonless Hayward, Minkowski core.

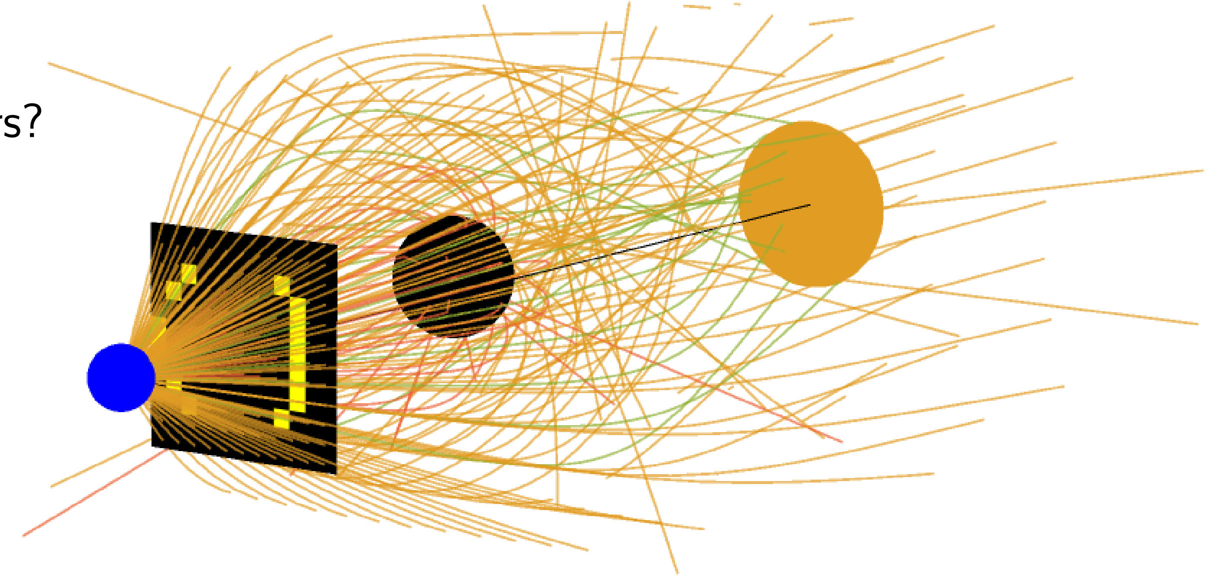
5/5 Conclusions

Conclusions / open problems / next steps

Gravitational microlensing has so far only been studied for black holes of general relativity.
But it can be a powerful tool to search for discrepancies in close-by events.

Open problems & next steps:

- How to extract new physics parameters?
- What about black hole rotation?
- Effects of black hole proper motion?
- Optimizations (adaptive mesh)?
- Extensions (radiative transfer)?
- ...



Thank you for your attention!

Gravitational microlensing and non-singular black holes

When a black hole passes in front of a luminous background source (like a star), the perceived brightness of the star temporarily increases due to gravitational lensing. Because the exact image of the star cannot typically be resolved exactly, this process is called 'microlensing.' This process has universal properties, but at shorter distances can be used to constrain the black hole metric directly. In this talk, we will first develop a numerical ray-tracing setup that allows us to depict the black hole and objects in its environments, and then describe how photometric lightcurves can be extracted using this method. We will close by comparing the results of general relativity to modified gravity models featuring non-singular black holes and find that non-singular black holes tend to have larger brightness maxima.