An exact stationary string configuration attached to a rotating black hole

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Stationary black holes with stringy hair

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Principal Killing strings in higher-dimensional Kerr-NUT-(A)dS spacetimes

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What are cosmic strings, and should we care?

The spontaneous breaking of a global symmetry G to a smaller group M = G / H can create topologically protected phases of non-zero, gauge-inequivalent vacuum expectation values of some matter field ϕ. (Kibble 1976)

Typical dimension of these defects is $\eta \sim \ell_p \frac{m_p}{m}$, and the typical tension is $\mu \sim \frac{m_p}{\ell_p}\left(\frac{m}{m_p}\right)^3$, where m = symmetry breaking scale. The approximation $\eta \approx 0$ corresponds to a 2D Nambu–Goto string.

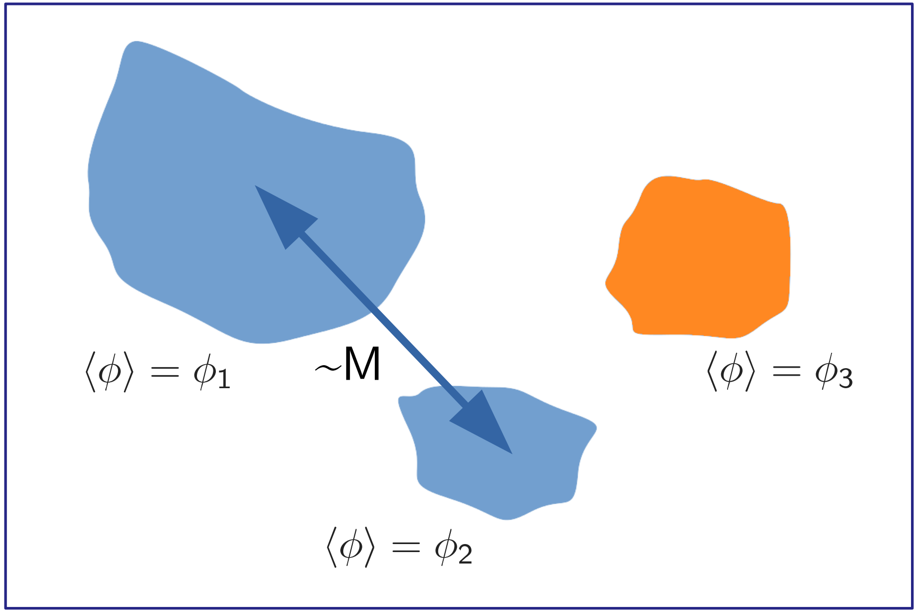

Fig. 1: Gauge-inequivalent vacuum expectation values form strings.

In general their interaction with black holes is time-dependent and can only be studied numerically. Our idea: understand some aspects of interaction with **black holes** using analytical techniques.

This is what the first snapshot of a rotating black hole could look like...

Figure by T. Bronzwaer, J. Davelaar, M. Moscibrodzka, and H. Falcke (Radboud University)

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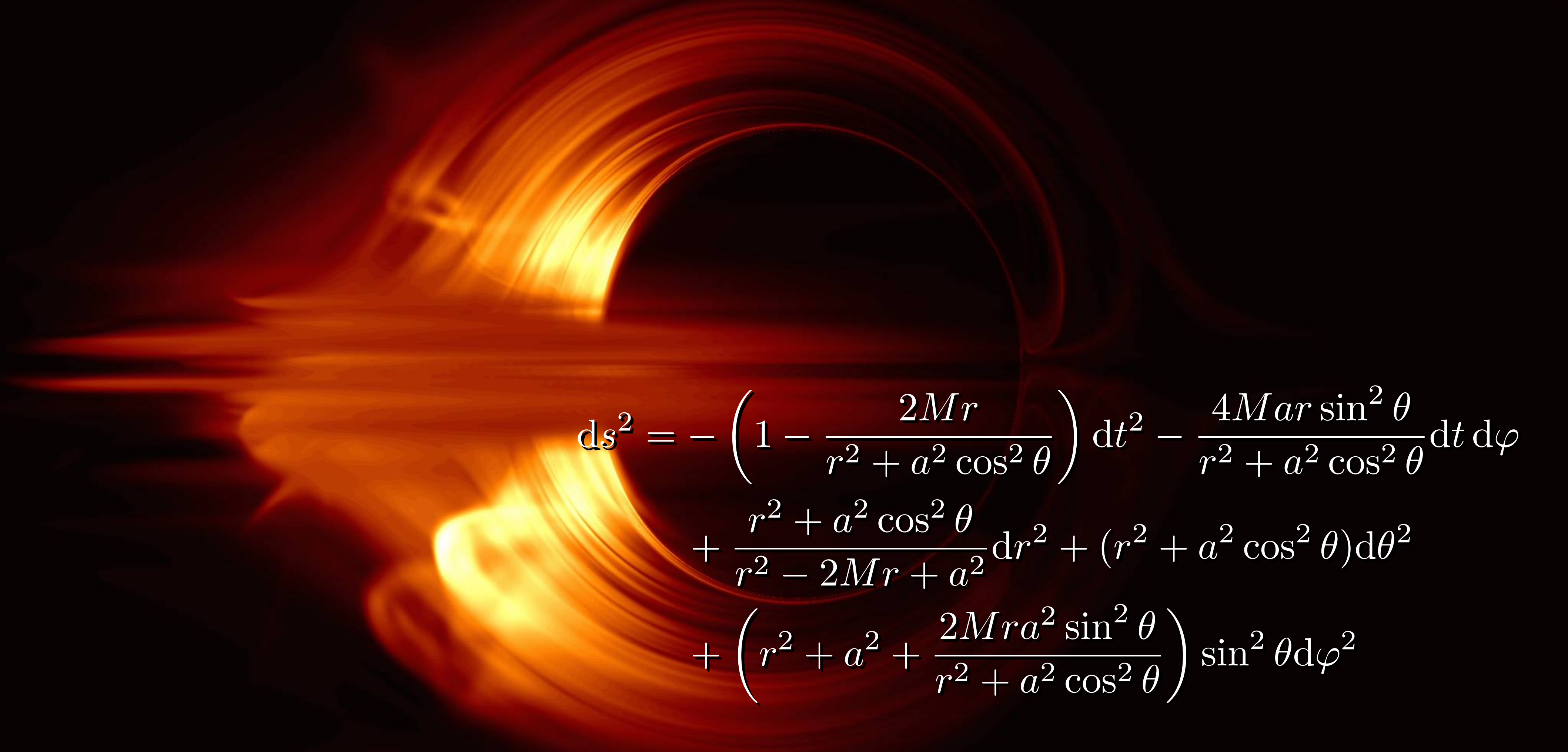


$$
\begin{aligned}
\mathrm{d}s^2 = &-\left(1 - \frac{2Mr}{r^2 + a^2\cos^2\theta}\right)\mathrm{d}t^2 - \frac{4Mar\sin^2\theta}{r^2 + a^2\cos^2\theta}\mathrm{d}t\,\mathrm{d}\varphi \\
&+ \frac{r^2 + a^2\cos^2\theta}{r^2 - 2Mr + a^2}\mathrm{d}r^2 + (r^2 + a^2\cos^2\theta)\mathrm{d}\theta^2 \\
&+ \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{r^2 + a^2\cos^2\theta}\right)\sin^2\theta\mathrm{d}\varphi^2
\end{aligned}
$$

Figure by T. Bronzwaer, J. Davelaar, M. Moscibrodzka, and H. Falcke (Radboud University)

Rotating black holes have very special properties

Rotating black holes (important for astrophysics) are described by the Kerr metric (see last slide). Mathematically, they are a specific subclass of a much larger class of geometries:

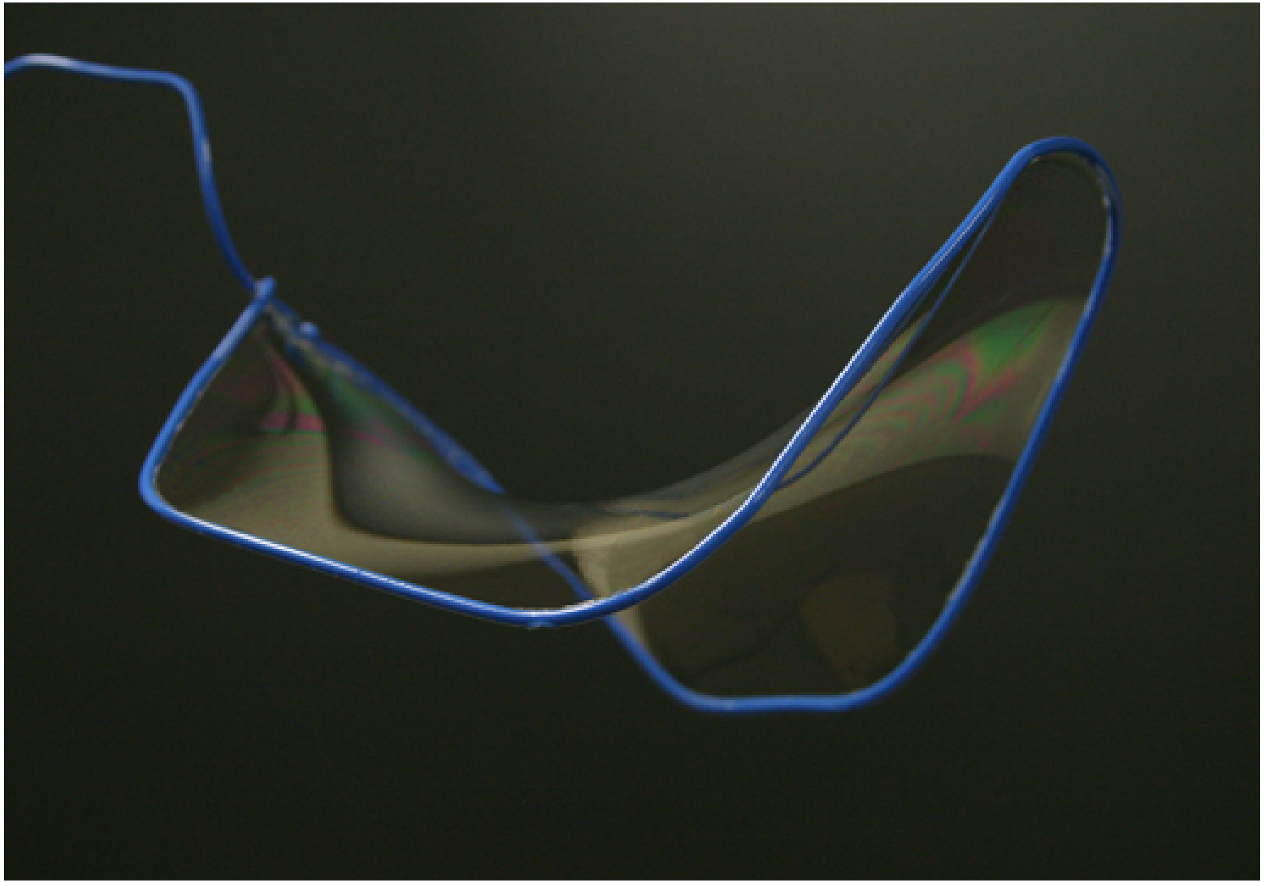
Kerr–NUT–(A)dS geometries are the most general Einstein spaces ($\mathrm{Ric}_{\mu\nu} \propto g_{\mu\nu}$) admitting a so-called conformal closed Killing–Yano tensor ($\to$ **principal tensor** $\boldsymbol{h_{\mu\nu} = -h_{\nu\mu}}$).

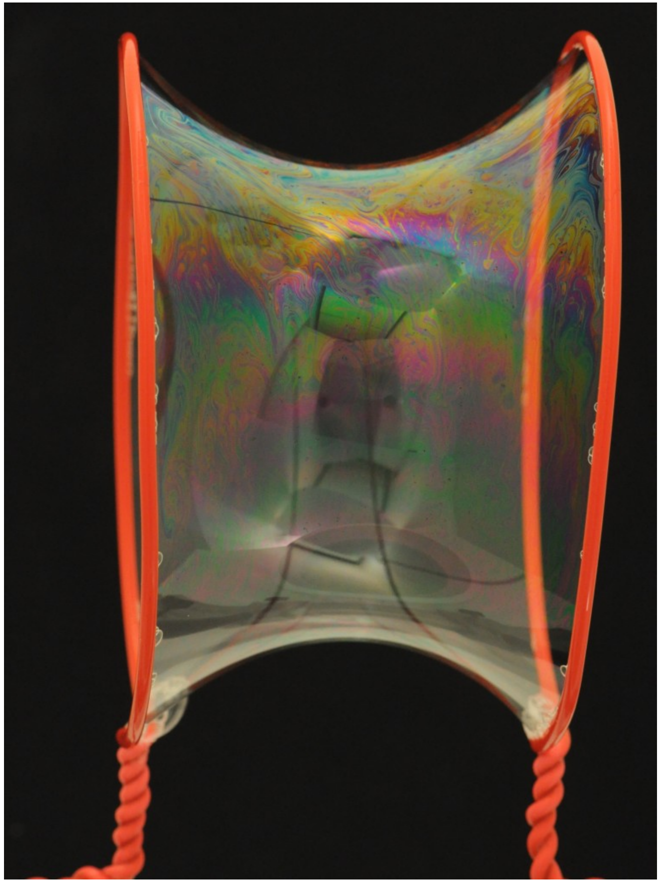
This principal tensor is important (and its existence quite unexpected)! It gives rise to

- primary Killing vector (= stationarity): $\xi^\mu = \frac{1}{D-1} \nabla_\nu h^{\nu\mu}$
- rank-2 Killing tensors (∼ Carter constant): $K_{\mu\nu} = f_{\mu\alpha\ldots} f_\nu{}^{\alpha\cdots}\,, \quad f = \star h$
- secondary Killing vectors (∼ axisymmetry): $\zeta^\mu = K^\mu{}_\nu \xi^\nu$

Let us use these properties of the principal tensor to construct a **stationary string configuration**.

Cosmic strings $\approx$ Nambu–Goto strings = minimal surfaces





http://www.math.hmc.edu/~jacobsen/demolab/soapfilm.html
http://www.soapbubble.dk/english/science/the-geometry-of-soap-films-and-soap-bubbles/

Our exact string configuration: “principal Killing strings”

For the Kerr spacetime, the string can be parametrized as

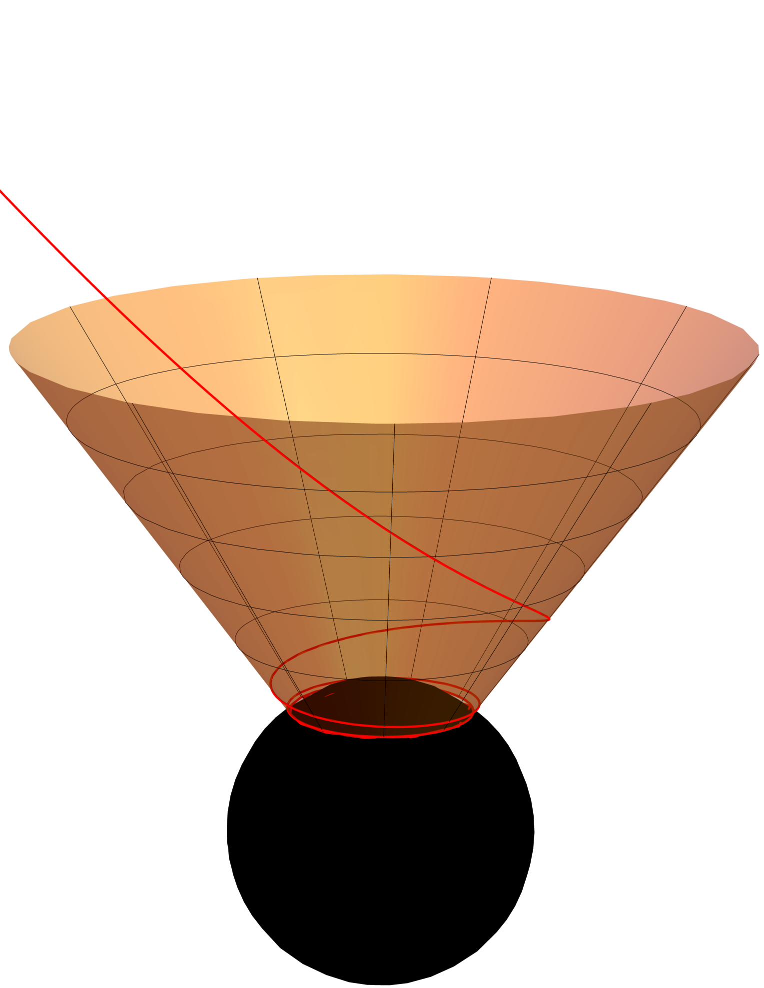
$$X^\mu(t,r) = \left(t, r, \theta_0, \phi_0 - \int \frac{a\,dr}{r^2 - 2Mr + a^2}\right) .$$

Important: it is tangent to the timelike Killing vector $\boldsymbol{\xi}$ as well as the (ingoing) principal null congruence $\boldsymbol{\ell}$.

It can be generalized to include a positive or negative cosmological constant, as well as to higher dimensions.

How to prove minimal surface?

⟶ Can show that trace of extrinsic curvature vanishes!



Proof: Principal Killing surfaces = minimal surfaces

Minimal surfaces have vanishing mean curvature, $\Omega_{(i)} \equiv \gamma^{AB}\Omega_{(i)AB} = 0$.

Rewrite as $\Omega_{(i)} = (\boldsymbol{n_{(i)}}, \boldsymbol{Z})\,, \quad Z^b = \gamma^{AB} Y^c{}_{,A} \nabla_c Y^b{}_{,B}$.

If we can show that $\boldsymbol{Z} \in T\Sigma$, we have proven that they are minimal surfaces.

Calculate: $Z^b = -\left(\xi^a \nabla_a \ell^b + \ell^a \nabla_a \xi^b + \boldsymbol{\xi}^2 \ell^a \nabla_a \ell^b\right) = -2\ell^a \nabla_a \xi^b \equiv -2 F_a{}^b \ell^a$

- Multiplying $\nabla_c h_{ab} = g_{ca}\xi_b - g_{cb}\xi_a$ by ξ^c implies that $\xi^c \nabla_c h_{ab} = 0$.
- Then, $\mathcal{L}_{\boldsymbol{\xi}} \boldsymbol{h} = 0$ becomes $F_a{}^b h_{bc} = F_c{}^b h_{ba}$.
- Also, know that $h^a{}_b \ell^b = -r\ell^a$.

Defining $V^a \equiv F^a{}_b \ell^b$ one has: $\longrightarrow$ $h^a{}_b V^b = h^a{}_b F^b{}_c \ell^c = F^a{}_b h^b{}_c \ell^c = -rV^a$

The non-degeneracy of h_{ab} then implies that $V^a \propto \ell^a$ which completes the proof.

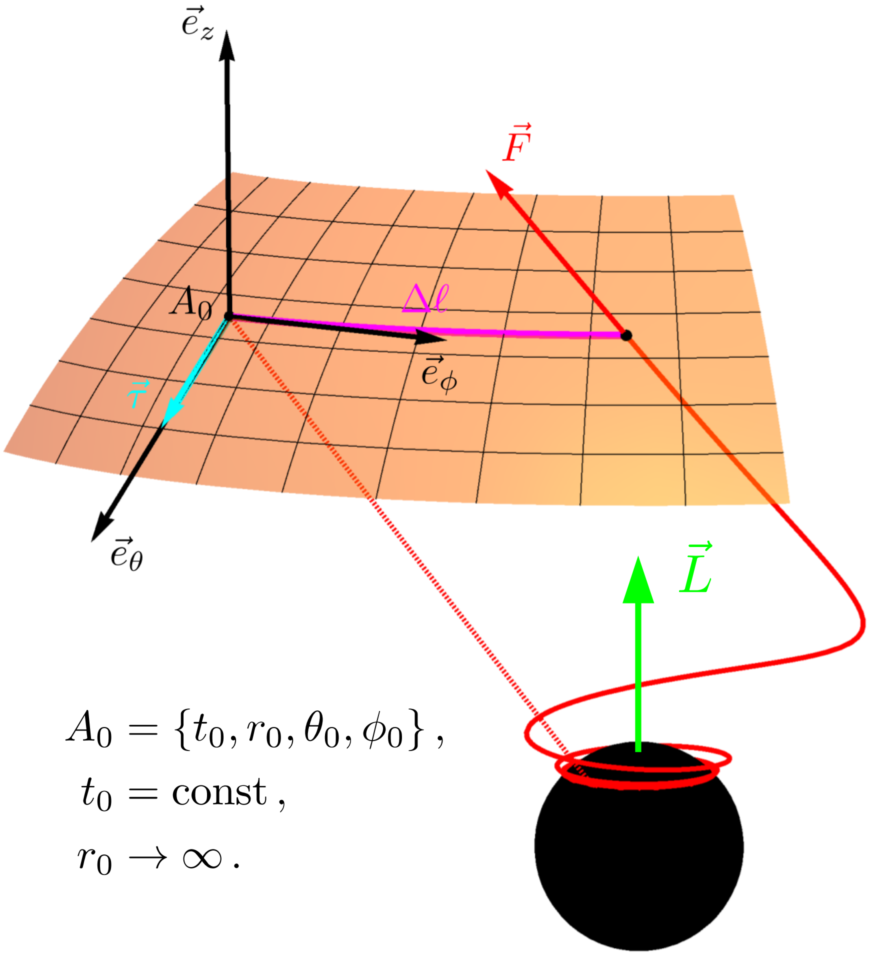
Principal Killing strings extract angular momentum from black holes.

Interesting physics in the (BH+string) system:

- The string pierces the black hole horizon, but the overall configuration is stationary.
- The string does not extract energy, rather, it extracts **angular momentum**.
- There is a simple mechanical interpretation:

$$\frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = \vec{\tau} = \Delta\vec{\ell} \times \vec{F}$$

Thank you for your attention.



$$A_0 = \{t_0, r_0, \theta_0, \phi_0\}\,,$$
$$t_0 = \mathrm{const}\,,$$
$$r_0 \to \infty\,.$$