Principal Killing strings in higher-dimensional Kerr–NUT–(A)dS geometries

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Stationary black holes with stringy hair

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Principal Killing strings in higher-dimensional Kerr-NUT-(A)dS spacetimes

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Kerr–NUT–(A)dS geometries

Kerr–NUT–(A)dS geometries are of Petrov type D, that is, they have two double principal null directions $\boldsymbol{\ell}$ that satisfy $\ell_{[e}C_{a]b[cd}\ell_{f]}\ell^b = 0$.

Kerr–NUT–(A)dS geometries are the most general Einstein spaces $(\mathrm{Ric}_{ab} \propto g_{ab})$ admitting a conformal closed Killing–Yano tensor (➞ **principal tensor**).

The principal tensor gives rise to

- primary Killing vector (= stationarity): $\xi^a = \frac{1}{D-1}\nabla_b h^{ba}$
- rank-2 Killing tensors (∼ Carter constant): $K_{ab} = f_{ai\ldots}f_b{}^{i\cdots}\,, \quad f = \star h$
- secondary Killing vectors (∼ axisymmetry): $\zeta^a = K^a{}_b\xi^b$

Let us use these properties of the principal tensor to construct a **stationary string solution**.

Ansatz: Principal Killing surfaces

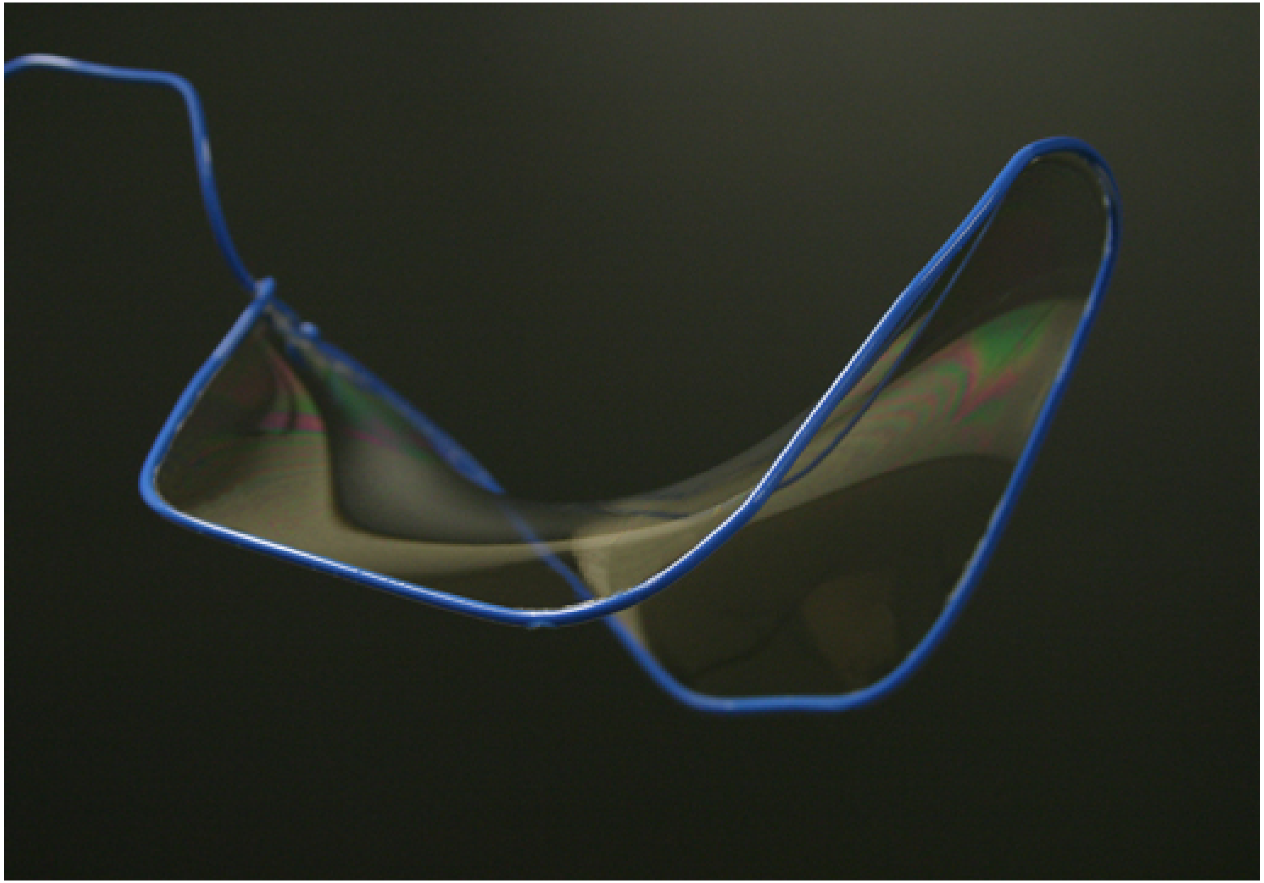
Let us denote by Σ a surface that is everywhere tangential to the two vectors $\boldsymbol{\xi}$ and $\boldsymbol{\ell}$. Because these vectors commute, $[\boldsymbol{\xi}, \boldsymbol{\ell}] = 0$, the Frobenius theorem allows for coordinates along the integral lines of these vector fields, $\boldsymbol{\xi} = \partial_v$ and $\boldsymbol{\ell} = \partial_\lambda$.

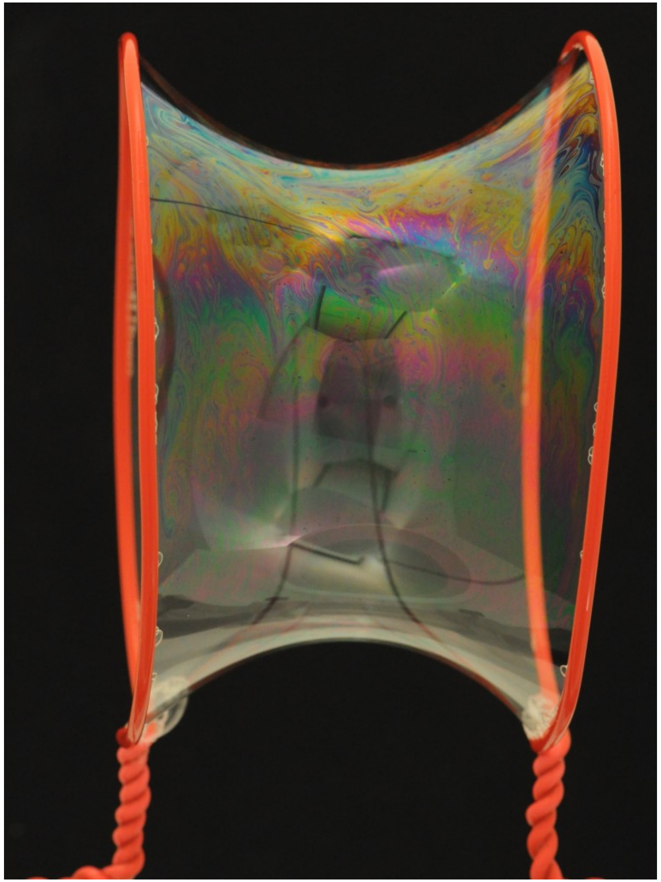
Then, the induced metric on Σ is given by $\mathrm{d}\gamma^2 = \boldsymbol{\xi}^2 \mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}\lambda$.

A parametrization of this surface is a function $Y^a = Y^a(v, \lambda)$ such that $Y^a{}_{,v} = \xi^a$ and $Y^a{}_{,\lambda} = \ell^a$. The integration constants correspond to angles (conical string that pierces the black hole horizon).

The extrinsic curvatures of this surface are then $\Omega_{(i)AB} = g_{ab} n^a_{(i)} Y^c{}_{,A} \nabla_c Y^b{}_{,B}$ for each normal $n^a_{(i)}$.

Claim: the properties of the principal tensor are enough to prove that Σ is a minimal surface.





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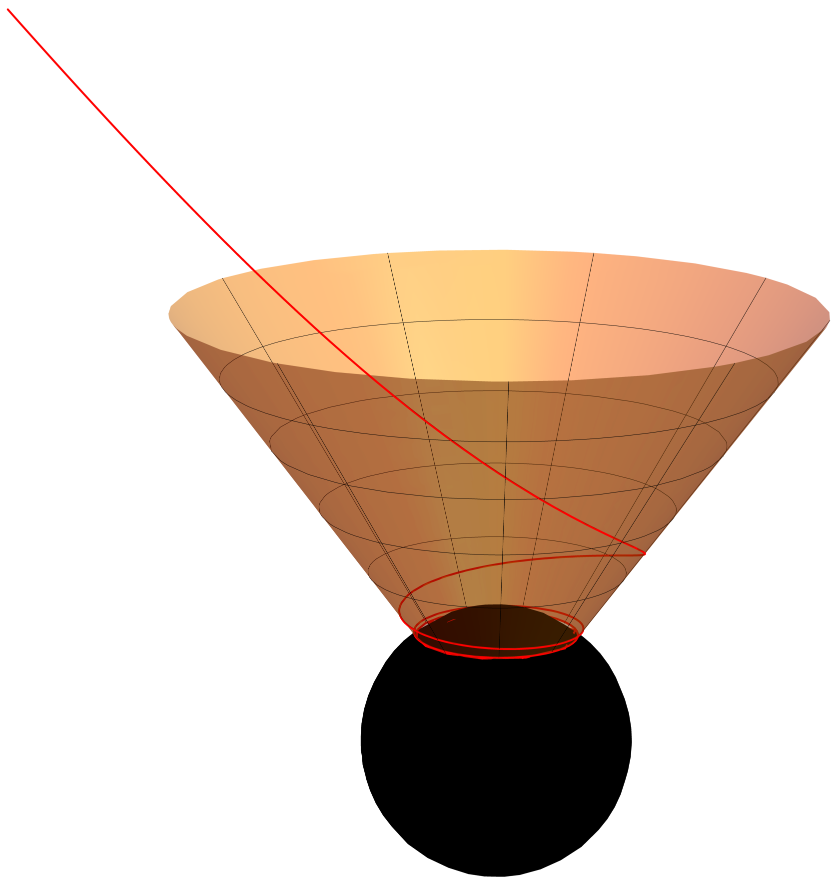


Proof: Principal Killing surfaces = minimal surfaces

Minimal surfaces have vanishing mean curvature, $\Omega_{(i)} \equiv \gamma^{AB}\Omega_{(i)AB} = 0$.
Rewrite as $\Omega_{(i)} = (\boldsymbol{n_{(i)}}, \boldsymbol{Z})$, $\quad Z^b = \gamma^{AB} Y^c{}_{,A} \nabla_c Y^b{}_{,B}$.
If we can show that $\boldsymbol{Z} \in T\Sigma$, we have proven that they are minimal surfaces.

Calculate: $Z^b = -\left(\xi^a \nabla_a \ell^b + \ell^a \nabla_a \xi^b + \boldsymbol{\xi}^2 \ell^a \nabla_a \ell^b\right) = -2\ell^a \nabla_a \xi^b \equiv -2 F_a{}^b \ell^a$

- Multiplying $\nabla_c h_{ab} = g_{ca}\xi_b - g_{cb}\xi_a$ by ξ^c implies that $\xi^c \nabla_c h_{ab} = 0$.
- Then, $\mathcal{L}_{\boldsymbol{\xi}} \boldsymbol{h} = 0$ becomes $F_a{}^b h_{bc} = F_c{}^b h_{ba}$.
- Also, know that $h^a{}_b \ell^b = -r\ell^a$.

Defining $V^a \equiv F^a{}_b \ell^b$ one has: $\rightarrow$ $h^a{}_b V^b = h^a{}_b F^b{}_c \ell^c = F^a{}_b h^b{}_c \ell^c = -rV^a$

The non-degeneracy of h_{ab} then implies that $V^a \propto \ell^a$ which completes the proof.

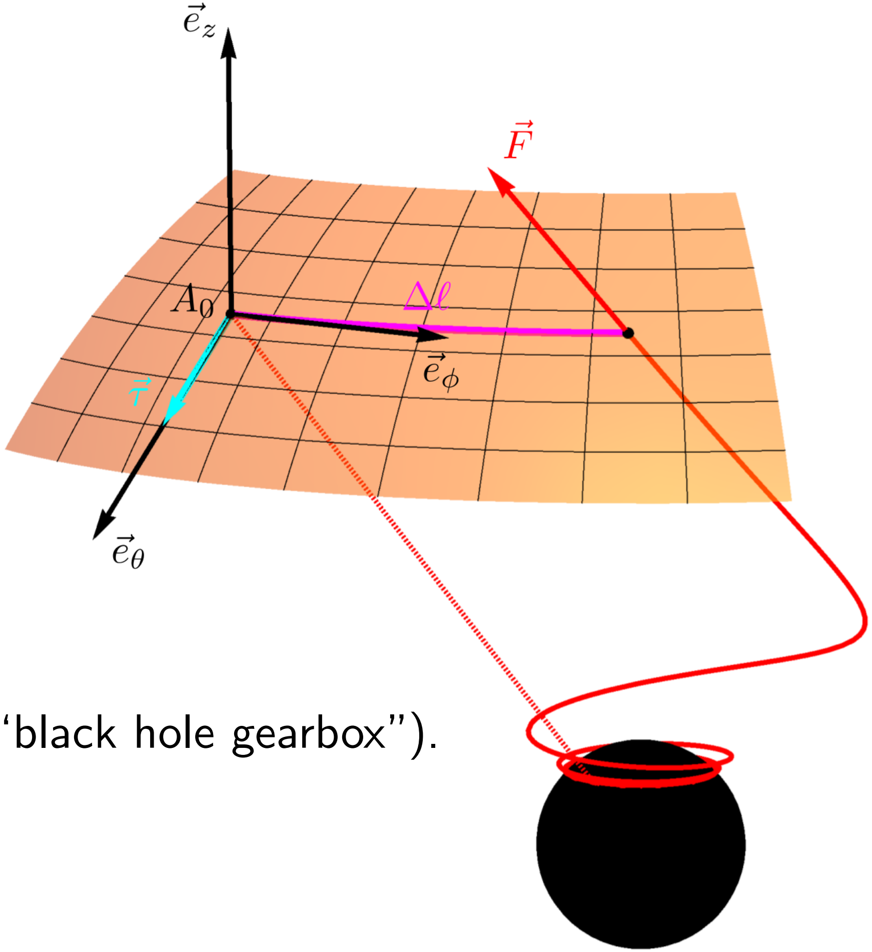
Principal Killing strings extract angular momentum from black holes.

Mass and angular momentum fluxes:

$$\Delta E = \int_S T^{ab} \xi_b \mathrm{d}\Sigma_a = 0\,,$$

$$\Delta J = \int_S T^{ab} \zeta_b \mathrm{d}\Sigma_a = -\mu_s a \sin^2\theta_0\,,$$

$$T^{ab} = \frac{\mu_s}{\sqrt{-g}} \left(\xi^{(a}\ell^{b)} + \boldsymbol{\xi}^2 \ell^a \ell^b \right) q\,.$$



Can attach multiple strings to remain stationary ("black hole gearbox").

Thank you for your attention.