Cosmic strings in stationary black hole geometries: stringy matter and principal Killing strings

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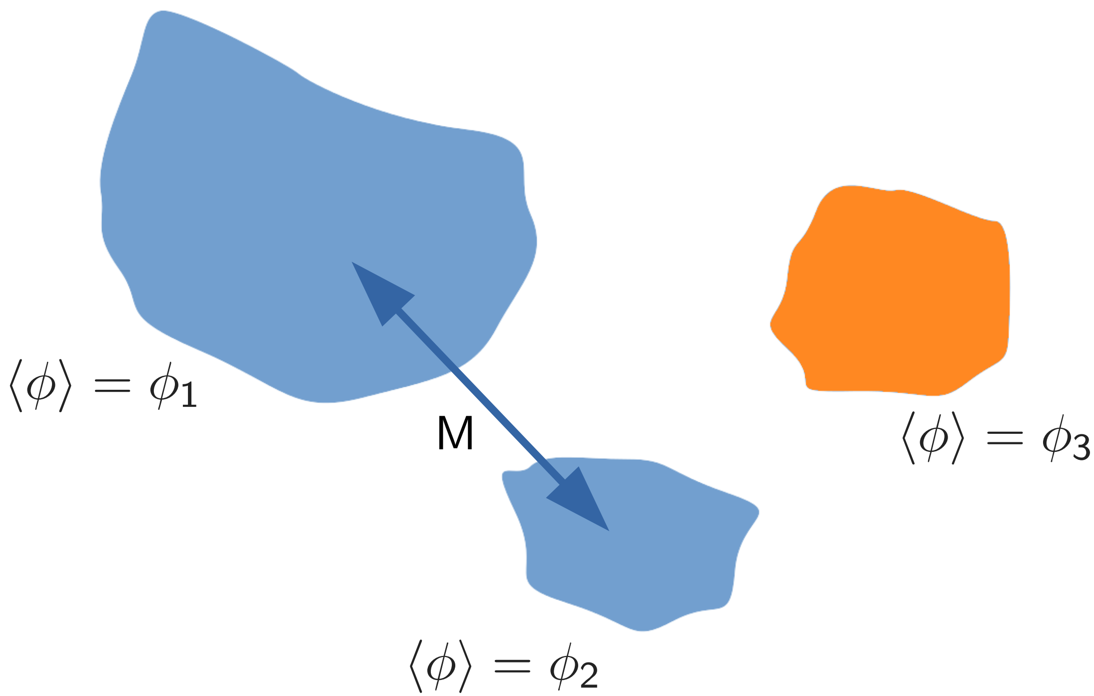
A cosmic string is a topological defect in spacetime.

The spontaneous breaking of a global symmetry G to a smaller group M = G / H can create topologically protected phases of non-zero, gauge-inequivalent vacuum expectation values of some matter field ϕ:



Typical dimension of these defects is $\eta \sim \ell_p \frac{m}{m_p}$, m = symmetry breaking scale.
The approximation $\eta \approx 0$ corresponds to a 1-dimensional string of zero width.

A cosmic string is a topological defect in solid continua.

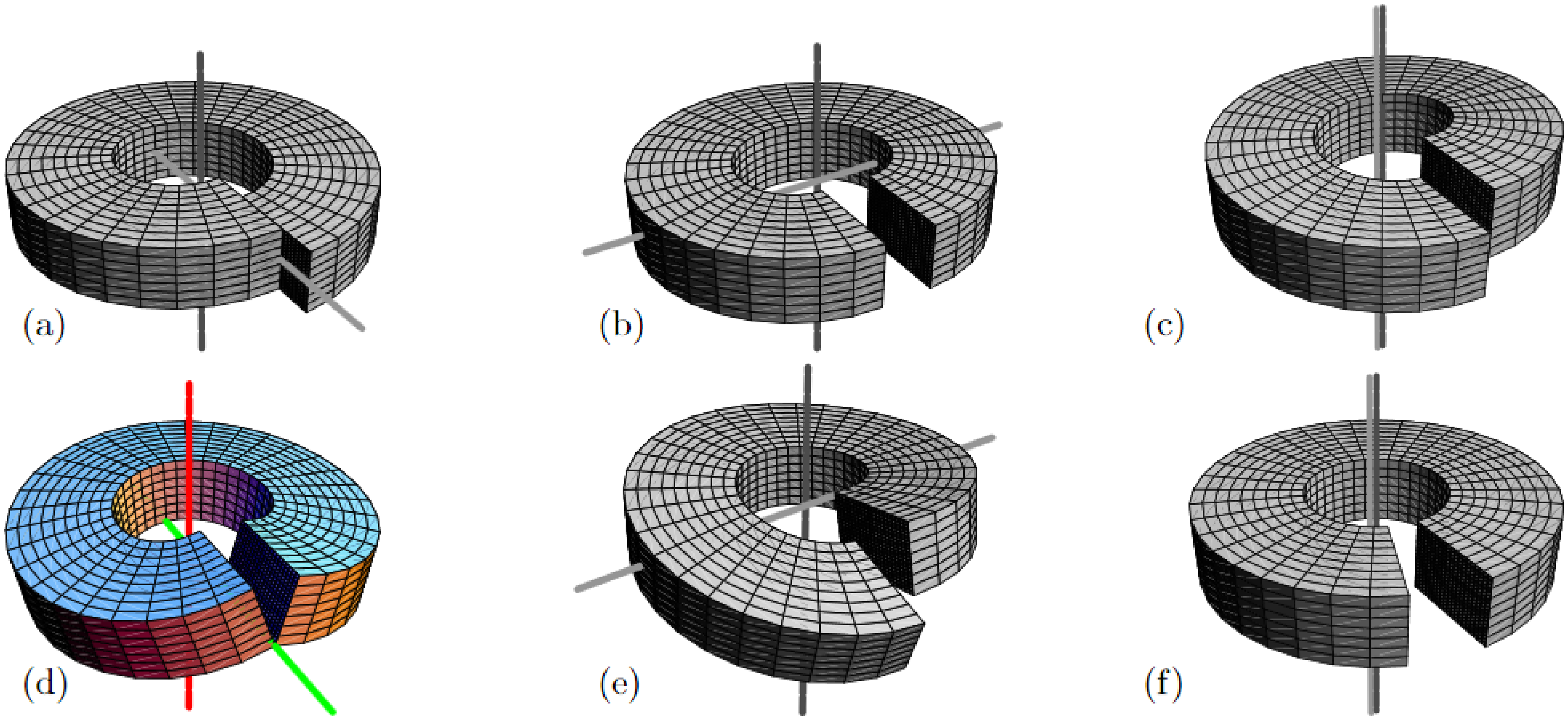


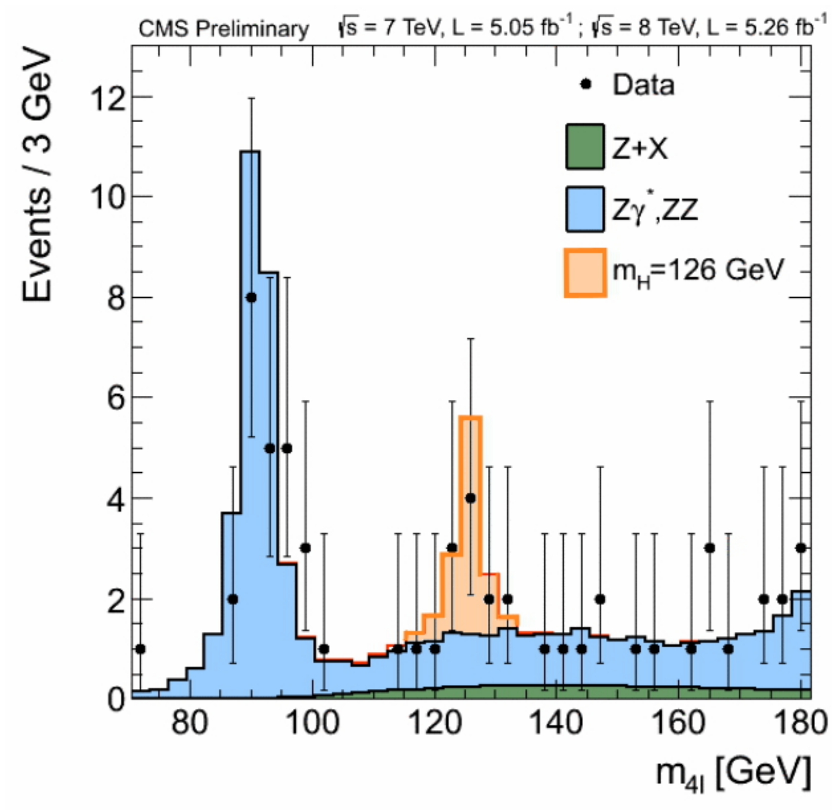
Figure 2: The six Volterra distortions. (a)–(c) Dislocations (order one to three). (d)–(f) Disclinations (order four to six)

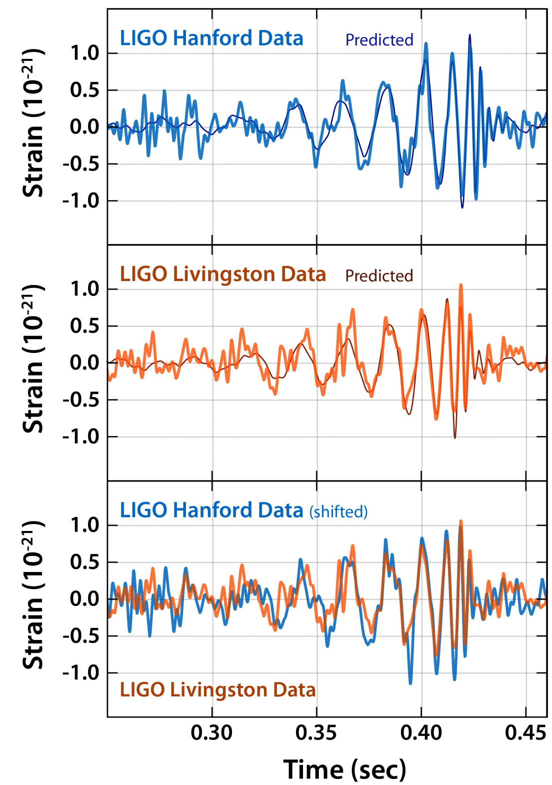
R. A. Puntigam and H. H. Soleng, "Volterra distortions, spinning strings, and cosmic defects," Class. Quant. Grav. **14** (1997) 1129, arXiv:gr-qc/9604057.

Cosmic strings in **stationary black hole geometries**:

stringy matter and principal Killing strings

Black holes have been detected.





Left: Preliminary results indicating the presence of a new boson at $m = 126$ GeV.

Right: Discovery of gravitational waves.

Black holes may become good laboratories.

How do astrophysical black holes interact with hypothesized physical objects?

- interaction of black holes with cosmic strings?

This problem is in general very complicated due to the time-dependence.

$\rightarrow$ consider instead static/stationary situations: $\xi \wedge \mathrm{d}\xi \sim \begin{cases} 0 & \text{: static} \\ \star\,\xi & \text{: stationary} \end{cases}$

In this talk, we consider two scenarios:

I. Stringy matter [J.B. and V. P. Frolov, 1711.06357]

- construct a static distribution of cosmic strings in the background of the static Reissner-Nordström-(A)dS black hole

II. Principal Killing strings [J.B. and V. P. Frolov, 1801.00122]

- construct a single stationary string using fundamental geometric properties of the Kerr–NUT–(A)dS spacetime

Cosmic strings in stationary black hole geometries:

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Strings source gravity.

Particles are 0-dimensional objects parametrized by $Y^a = Y^a(z)$, a=0,1,2,3.
Strings are 1-dimensional objects parametrized by $Y^a = Y^a(z^A)$, a=0,1,2,3, A=1,2.

The string parametrization ("worldsheet") minimizes the Nambu–Goto action

$$S = -\mu \int d^2z \sqrt{-\gamma}\,, \quad \gamma_{AB} = g_{ab} Y^a{}_{,A} Y^b{}_{,B}\,, \quad \mu = \text{ tension } = \frac{\text{force}}{\text{length}}\,.$$

Strings couple to gravity via the stress-energy tensor

$$T^{ab} = \frac{\mu}{\sqrt{-g}} \int d^2z \sqrt{-\gamma}\gamma^{AB} Y^a{}_{,A} Y^b{}_{,B}\, \delta^{(4)}(x^a - Y^a(z))\,.$$

For a straight string along the z-axis:

$$T^a{}_b = -\mu\delta(x)\delta(y) \underbrace{\mathrm{diag}(1,0,0,1)}_{\substack{\text{equation of state} \\ \text{“stringy matter”}}}\,, \quad \{0,1,2,3\} = \{t,x,y,z\}\,.$$

Stringy matter deforms a static black hole.

Exact result: a spherical distortion of the Reissner-Nordström-(A)dS metric (G=c=1)

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 e^{2\sigma(\theta,\phi)} \left(d\theta^2 + \sin^2\theta d\phi^2\right) \ ,$$

$$f = 1 - \frac{2m}{r} + \frac{q^2}{r^2} - \frac{1}{3}\Lambda r^2 \quad A = \frac{q}{r} dt \, , \quad F = dA \, ,$$

is a solution of the Einstein–Maxwell equations with cosmological constant

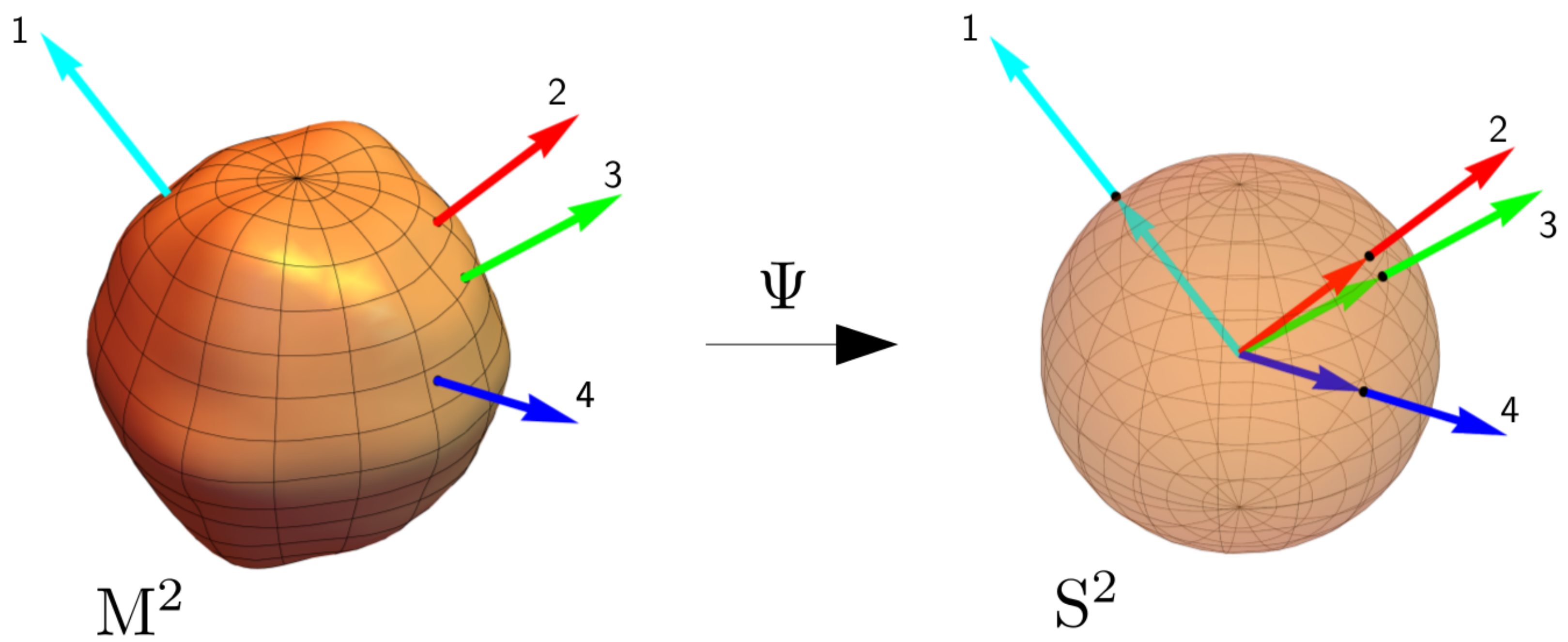
$$R^a{}_b - \frac{1}{2} R \delta^a_b + \Lambda \delta^a_b = 8\pi T^a{}_b \, , \quad F^{ab}{}_{;b} = 0 \, ,$$

with the stress-energy tensor (and $\{0,1,2,3\} = \{t, r, \theta, \phi\}$):

$$\begin{aligned} T^a{}_b &= \frac{q^2}{8\pi r^4} \mathrm{diag}\big(-1, -1, 1, 1\big) \quad + \quad \frac{\Phi}{8\pi r^2} \mathrm{diag}\big(1, 1, 0, 0\big) \\ &= \frac{1}{4\pi} \left(F^{ac} F_{bc} - \frac{1}{4} \delta^a_b F_{cd} F^{cd} \right) \ + \ \text{“stringy matter”} \end{aligned}$$

$$\Phi = 1 - e^{-2\sigma} \left(1 - \Delta\sigma\right) \ .$$

Gauss–Bonnet theorem serves as a consistency condition.



The deformed 2-sphere (left) and the 2-sphere (right) are related via the Gauss map Ψ.

The deformed black hole is static iff the "net force" acting via the strings vanishes:

$$\vec{F} \sim \int_{M^2} \mathrm{d}A\, \Phi\, \vec{n} = 0 \quad \leftrightarrow \text{ guaranteed by Gauss–Bonnet theorem}$$

Note: single string changes topology of the 2-sphere $\rightarrow$ net force no longer vanishes.

Cosmic strings in stationary black hole geometries:
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There are stationary, conical strings in the Kerr spacetime.

These strings have the parametrization

$$Y^a(t, r) = \left(t, r, \theta_0, \phi_0 - \int \frac{a\,dr}{\Delta}\right) ,$$

$$\Delta := r^2 - 2Mr + a^2 .$$

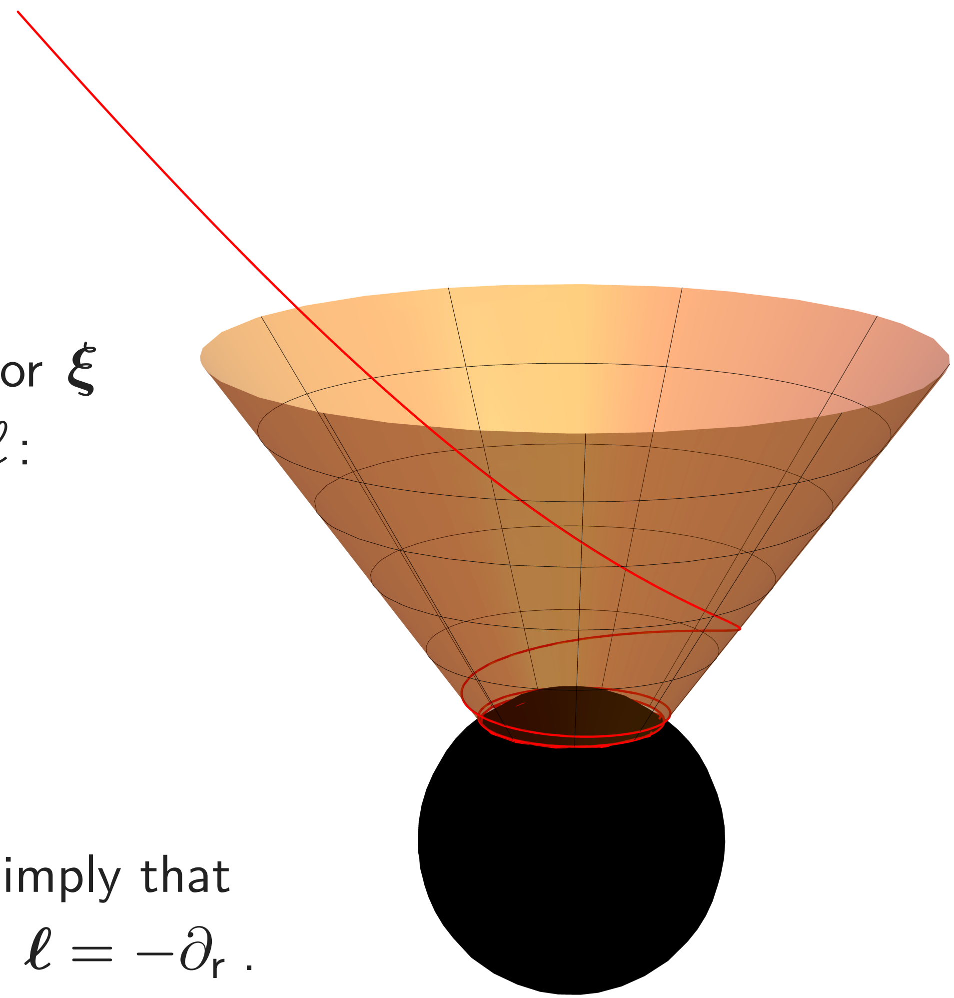
They are tangent to the timelike Killing vector $\boldsymbol{\xi}$ and the (ingoing) principal null congruence $\boldsymbol{\ell}$:

$$\partial_t Y = \boldsymbol{\xi} = \partial_t ,$$

$$\partial_r Y = -\boldsymbol{\ell} = -\partial_r + \frac{a}{\Delta}\partial_\phi .$$

Then, $[\boldsymbol{\xi}, \boldsymbol{\ell}] = 0$ and the Frobenius theorem imply that there are coordinates such that $\boldsymbol{\xi} = \partial_v$ and $\boldsymbol{\ell} = -\partial_r$.

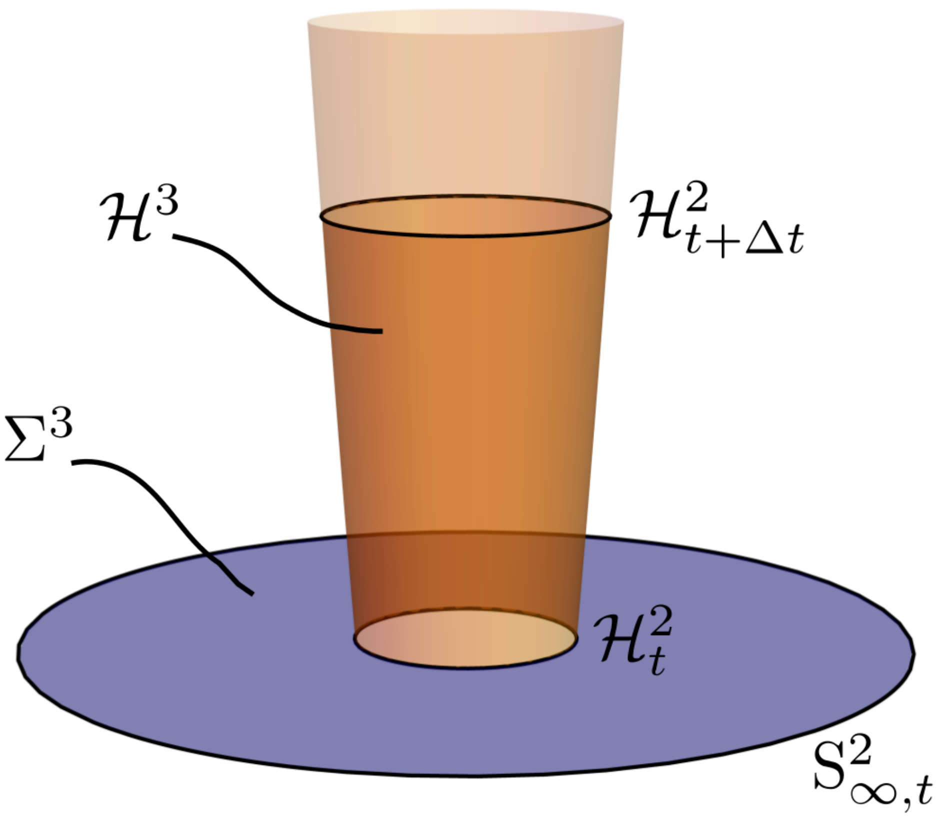
The induced metric is then $d\gamma^2 = \boldsymbol{\xi}^2 dv^2 + 2dvdr$, and one can show that it describes a minimal surface.



Conical strings extract angular momentum.

The Komar integrals for the mass M and angular momentum J of the Kerr spacetime:

$$-8\pi M_t^\infty = \oint_{S^2_{\infty,t}} \nabla^a \xi^b \mathrm{d}S_{ab}\,, \quad -16\pi J_t^\infty = \oint_{S^2_{\infty,t}} \nabla^a \zeta^b \mathrm{d}S_{ab}\,.$$

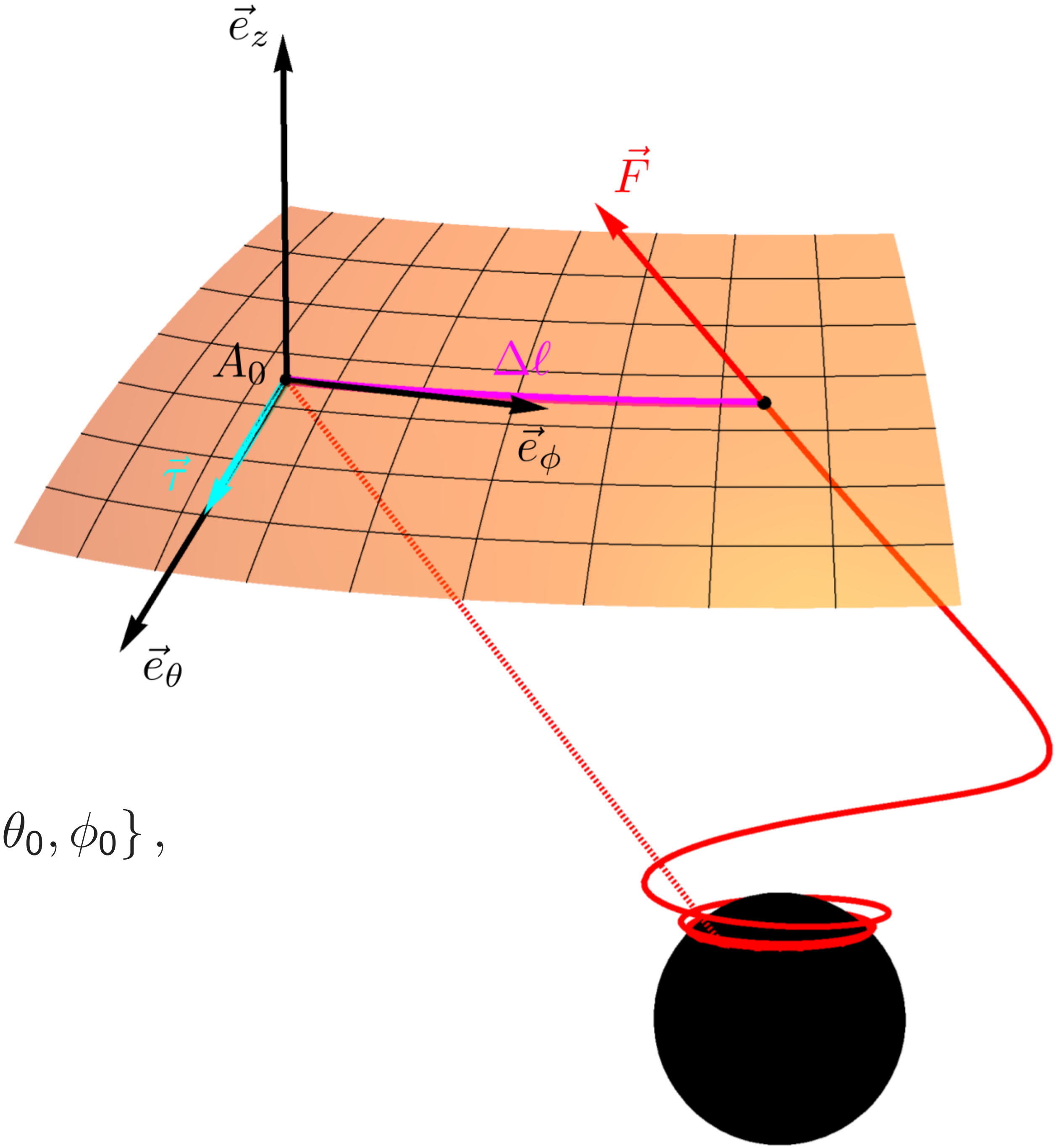


Any Killing vector **K** satisfies $\nabla^a \nabla_a K^b = -R^b{}_a K^a$, which can be used to integrate by parts.

The presence of $T^a{}_b$ leads to

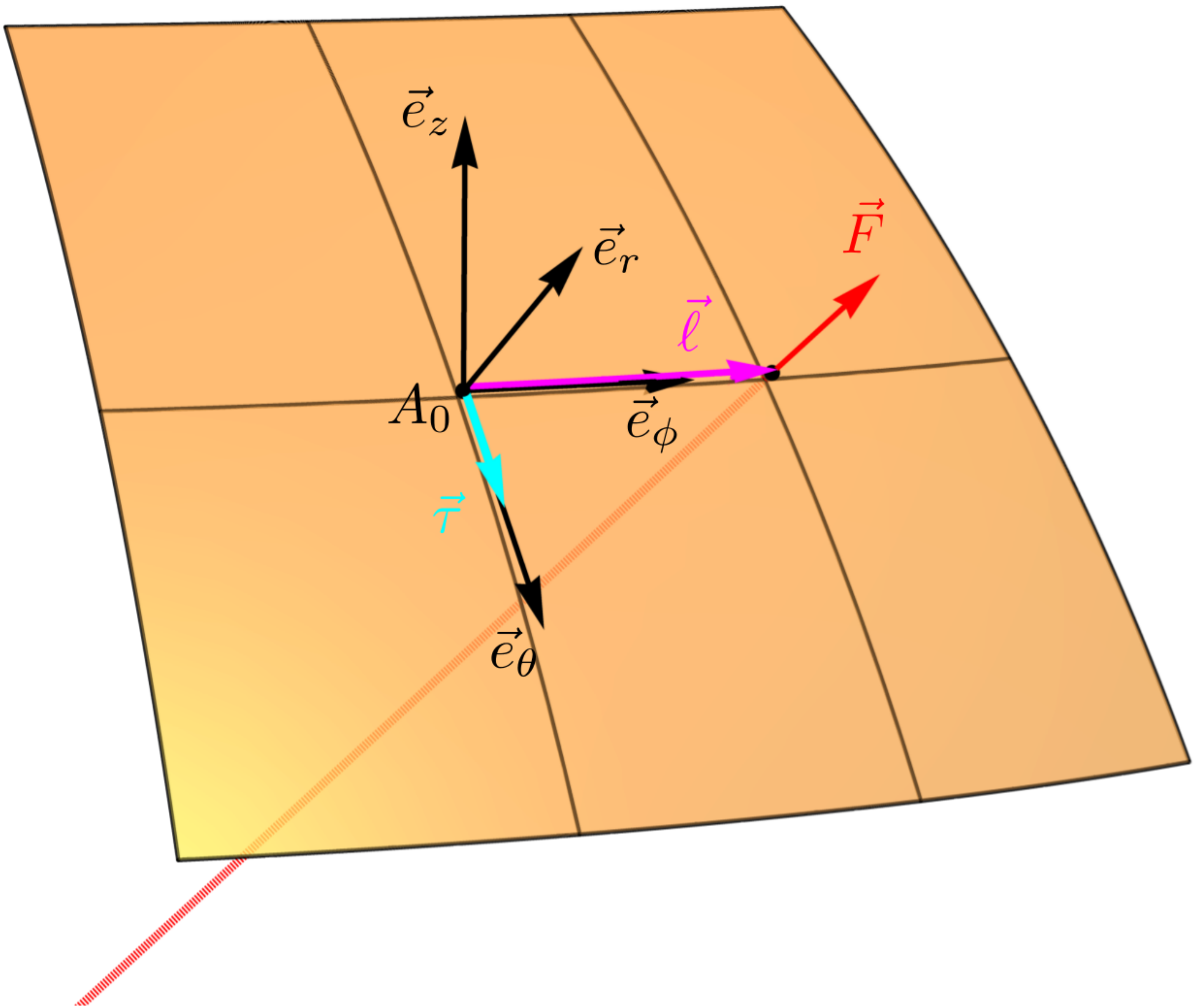
$$\dot{M}_\infty = 0\,,$$
$$\dot{J}_\infty = -\mu a \sin^2\theta\,.$$

Angular momentum is extracted by an asymptotic torque.

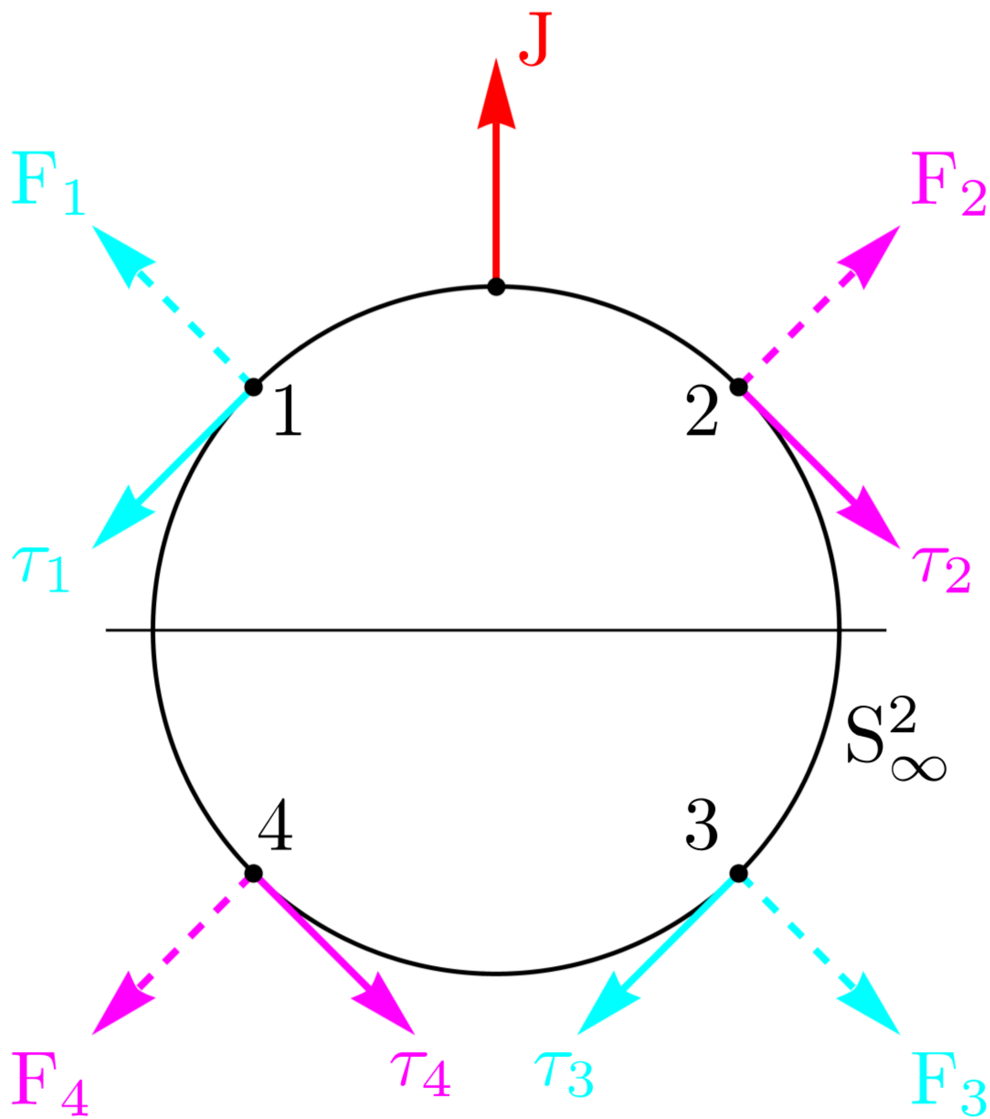


$$A_0 = \{t_0, r_0, \theta_0, \phi_0\}\,,$$
$$t_0 = \text{const}\,,$$
$$r_0 \to \infty\,.$$

Angular momentum is extracted by an asymptotic torque.



Adding strings can align the torque to the angular momentum.



Principal Killing strings in higher-dimensional Kerr–NUT–(A)dS

Kerr–NUT–(A)dS spacetimes in any dimension admit the non-degenerate closed conformal Killing–Yano 2-form **h** which satisfies .

$$\nabla_a h_{bc} = g_{ab}\xi_c - g_{ac}\xi_b \,, \quad \xi^a \equiv \frac{1}{D-1}\nabla_b h^{ba} \,.$$

Remarkable fact: this **principal tensor** is enough to construct "principal Killing strings."

- $h^a{}_b \ell^b_\pm = \mp r \ell^a_\pm$: principal null congruences are eigenvectors
- $[\boldsymbol{\xi}, \boldsymbol{\ell}_\pm] = 0$: we can use the Frobenius theorem
- $(\nabla_b \xi^a)\ell^b_\pm = \kappa_\pm \ell^a_\pm$: principal Killing strings are minimal surfaces

The stress-energy tensor of a principal Killing string is given by

$$T^{ab} = -\frac{\mu}{\sqrt{-g}}\left(2\xi^{(a}\ell^{b)} + \boldsymbol{\xi}^2 \ell^a \ell^b\right) q(\theta_0, \phi_0) \,.$$

Conclusions

Cosmic strings are of experimental and theoretical interest.

Static black holes and cosmic strings

- Stringy matter automatically respects staticity via the Gauss–Bonnet theorem.
- The notion of stringy matter might be an effective description of new physics.
- There are interesting topological applications.

Static black holes and principal Killing strings

- Only specially aligned strings respect the stationarity of the Kerr spacetime
- The principal tensor lies at hears of many analytical results for the entire class of higher-dimensional Kerr–NUT–(A)dS geometries.

Thank you for your attention.

Abstract

Cosmic strings are interesting objects both from an experimental and a theoretical point of view. In this talk, we discuss two scenarios:

(i) The stress-energy tensor of a string without internal structure satisfies a simple equation of state (“stringy matter”). We demonstrate that spherical deformations of the Reissner-Nordström-(A)dS metric give rise to such stringy matter, which in turn can be interpreted as a continuous distribution of cosmic strings.

(ii) For stationary spacetimes admitting a non-degenerate closed conformal Killing-Yano 2-form (e.g. the Kerr-NUT-(A)dS geometry), we construct a special stationary string configuration from the principal null congruence and the timelike Killing vector. In the special case of the Kerr metric (and higher-dimensional generalizations thereof) these strings extend from the black hole horizon to spatial infinity and extract angular-momentum from the black hole. We interpret this as the action of an asymptotic torque.

References

[1] J.B. and V. P. Frolov, “Stationary black holes with stringy hair,” arXiv:1711.06357 [gr-qc].

[2] J.B. and V. P. Frolov, “Principal Killing strings in higher-dimensional Kerr-NUT-(A)dS spacetimes,” arXiv:1801.00122 [gr-qc].