Effects of Non-Locality in Gravity and Quantum Theory — Ph.D. Defense

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Chair: Prof. Richard Marchand
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Wednesday, September 16, 2020
2:00pm, University of Alberta, Edmonton, online seminar.

Major open problems in gravitational physics

We live in a fascinating time: observation of gravitational waves & black holes, precision cosmology. So far, General Relativity has proven to be a remarkably accurate description of gravity.

strengthening of General Relativity $\Rightarrow$ take its problematic predictions more seriously!

There are several long-standing major open problems in gravitational physics:

- Black holes contain singularities (and may also contain Cauchy horizons).
- Black hole information loss paradox.
- Our entire Universe is thought to evolve from a cosmological singularity ("big bang").

It is conceivable that (some of) these problems can be solved by **new physics** that becomes relevant at very small distances and high energies, perhaps in an effective field theory of quantum gravity.

Our new ingredient: fundamental Lorentz-invariant ghost-free non-locality

We ask: what if Nature is fundamentally **non-local*** at a small length (or time) scale ℓ ?
*What do we mean by non-locality?

- Non-locality as a fundamental property of Nature, different from effective actions.
- Lorentz-invariant non-locality, unlike modified dispersion relations in condensed matter, and unconstrained from Lorentz-violating experiments.
- Ghost-free non-locality is not mediated by additional degrees of freedom, unlike generic higher-derivative theories, and unconstrained from thermodynamical history of the Universe.

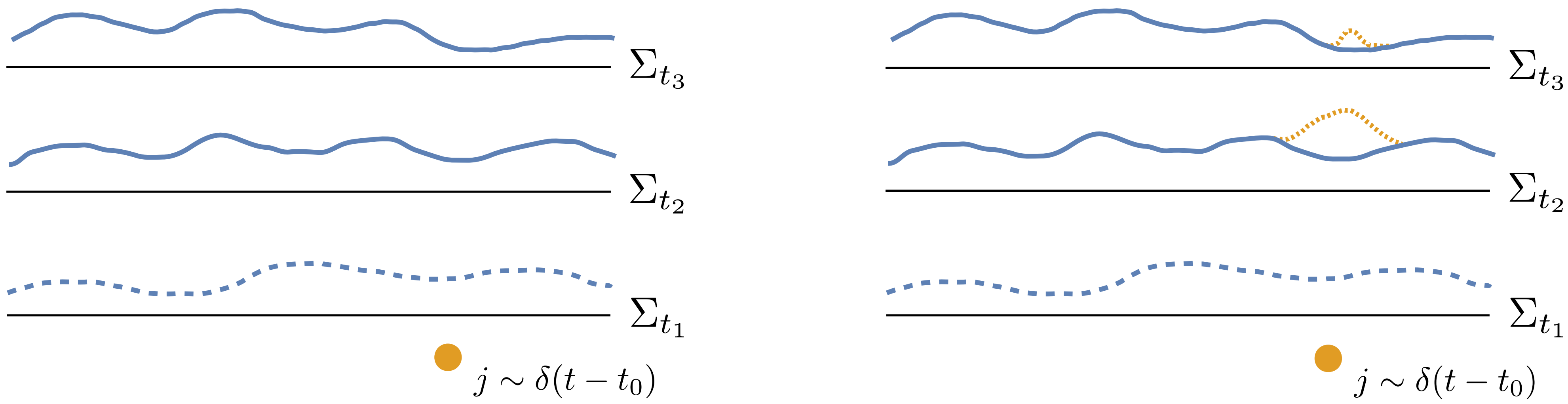
Goal: Understand this in the strong-field regime at the full non-linear level. This is very difficult.

Approach: study the weak-field regime of this kind of non-locality and develop **new tools** to do so.
Today's goal: Build some analytical non-local intuition in both gravity and quantum theory.
There are **many unexpected results** and **effects of non-locality**, and here I will report on them.

Thesis is based on these publications

8. JB, J. Pinedo Soto, and V. P. Frolov, "Ultrarelativistic spinning objects in non-local ghost-free gravity," Phys. Rev. D **101** (2020) no. 12, 124065; arXiv:2004.07420 [gr-qc].
7. JB, "Angle deficit & non-local gravitoelectromagnetism around a slowly spinning cosmic string," arXiv:2003.13847 [gr-qc]; GRF honorable mention 2020; invited for publication in Int. J. Mod. Phys. D
6. JB, V. P. Frolov, and A. Zelnikov, "Ghost-free modification of the Polyakov action and Hawking radiation," Phys. Rev. D **100** (2019) no. 10, 104008; arXiv:1909.01494 [hep-th].
5. JB, V. P. Frolov, and A. Zelnikov, "On thermal field fluctuations in ghost-free theories," Phys. Lett. B **793** (2019) 290 ; arXiv:1904.07917 [hep-th].
4. JB, V. P. Frolov, and A. Zelnikov, "Probing the vacuum fluctuations in scalar ghost-free theories," Phys. Rev. D **99**, no. 7 (2019) 076014; arXiv:1901.07096 [hep-th].
3. JB, V. P. Frolov, and A. Zelnikov, "Quantum scattering on a delta potential in ghost-free theory," Phys. Lett. B **782** (2018) 688; arXiv:1805.01875 [hep-th].
2. JB, "Gravitational Friedel oscillations in higher-derivative and infinite-derivative gravity?," Int. J. Mod. Phys. D **27** (2018) 1847022; GRF honorable mention 2018; arXiv:1804.00225 [gr-qc]
1. JB, V. P. Frolov, and A. Zelnikov, "Gravitational field of static p-branes in linearized ghost-free gravity," Phys. Rev. D **97**, no. 8 (2018) 084021; arXiv:1802.09573 [gr-qc].

Why not just use numerics? The necessity for new tools.



The local initial value problem (left) cannot be simply extended to the non-local case (right).

- Non-locality can lead to violations of causality.
- Well-established notions (Cauchy surface) lose their relevance in non-local theories.

New methods and tools are required to understand this class of non-local theories!

Let us focus on linear theories and develop the notion of **non-local Green functions**.

Non-local Green functions

Our model of non-locality: non-local form factors $f(\Box) = \exp\left[(-\ell^2\Box)^N\right]\,,\ N = 1, 2, \ldots\,.$

There is a huge variety of theories, but the above allows us to capture some general features.

General requirement: $1/f(\Box)$ must not have poles in the complex plane.

An example for a linear non-local equation is $f(\Box)\Box\varphi(x) = j(x)$.

Non-local Green functions $\mathcal{G}(x', x)$ generate solutions of these non-local differential equations.

$$\mathcal{G}(x', x) = \frac{1}{f(\Box)\Box} = G(x', x) + \Delta\mathcal{G}(x', x) \quad = \quad \text{local part} + \textbf{new, non-local part}\ .$$

In special situations these Green functions can be calculated, and we can study their properties.

Main focus: flat spacetime. Can study their Fourier decomposition and restrict to static cases.

Now: application to weak-field gravity and linear quantum theory.

Express **new effects** in terms of scale of non-locality ℓ to build non-local intuition.

Effects of Non-locality in Gravity and Quantum Theory

Weak-field ghost-free gravity: Lagrangian and field equations [Chs. 2 & 3]

Consider a small perturbation around Minkowski space, $g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}$ with $\epsilon \ll 1$.

$$
\begin{aligned}
S[h_{\mu\nu}] &= \frac{1}{2\kappa}\int \sqrt{-g}\mathrm{d}^D x \left(R + \frac{1}{2} R_{\mu\nu\rho\sigma} \mathcal{O}^{\mu\nu\rho\sigma}_{\alpha\beta\gamma\delta}(\nabla, \Box) R^{\alpha\beta\gamma\delta} \right) \\
&= \frac{1}{2\kappa}\int \mathrm{d}^D x \Big(\quad \frac{1}{2} h^{\mu\nu}\, \boldsymbol{a}(\Box)\Box\, h_{\mu\nu} - h^{\mu\nu}\, \boldsymbol{a}(\Box)\partial_\mu\partial_\alpha\, h^{\alpha}{}_{\nu} + h^{\mu\nu}\, \boldsymbol{c}(\Box)\partial_\mu\partial_\nu h \\
&\qquad - \frac{1}{2} h\, \boldsymbol{c}(\Box)\Box h + \frac{1}{2} h^{\mu\nu}\, \frac{\boldsymbol{a}(\Box) - \boldsymbol{c}(\Box)}{\Box} \partial_\mu\partial_\nu\partial_\alpha\partial_\beta\, h^{\alpha\beta} \Big) + \mathcal{O}(\epsilon^3)\,.
\end{aligned}
$$

Let us set the two **form factors** equal, $\boldsymbol{a}(\Box) = \boldsymbol{c}(\Box)$, and then the linear field equations are

$$
a(\Box)\Box\hat{h}_{\mu\nu} = -2\kappa T_{\mu\nu}\,, \quad \hat{h}_{\mu\nu} = \hat{h}^0_{\mu\nu} + 2\kappa \int \mathrm{d}^D y\, \mathcal{G}(\boldsymbol{x}, \boldsymbol{y}) T_{\mu\nu}(\boldsymbol{y})\,.
$$

Given $T_{\mu\nu}$, this can be solved with the Green function method and reduces to Laplace equation.

Weak-field ghost-free gravity: regular point particles and extended objects

Resulting non-local Poisson equation for a point particle:

$$\nabla^2 e^{-\ell^2\nabla^2}\phi = 4\pi G m \delta^{(3)}(\boldsymbol{r}) \quad \longrightarrow \quad \phi(r) = -4\pi G m \mathcal{G}_3(r) = \frac{-Gm}{r}\mathrm{erf}\left(\frac{r}{2\ell}\right)$$

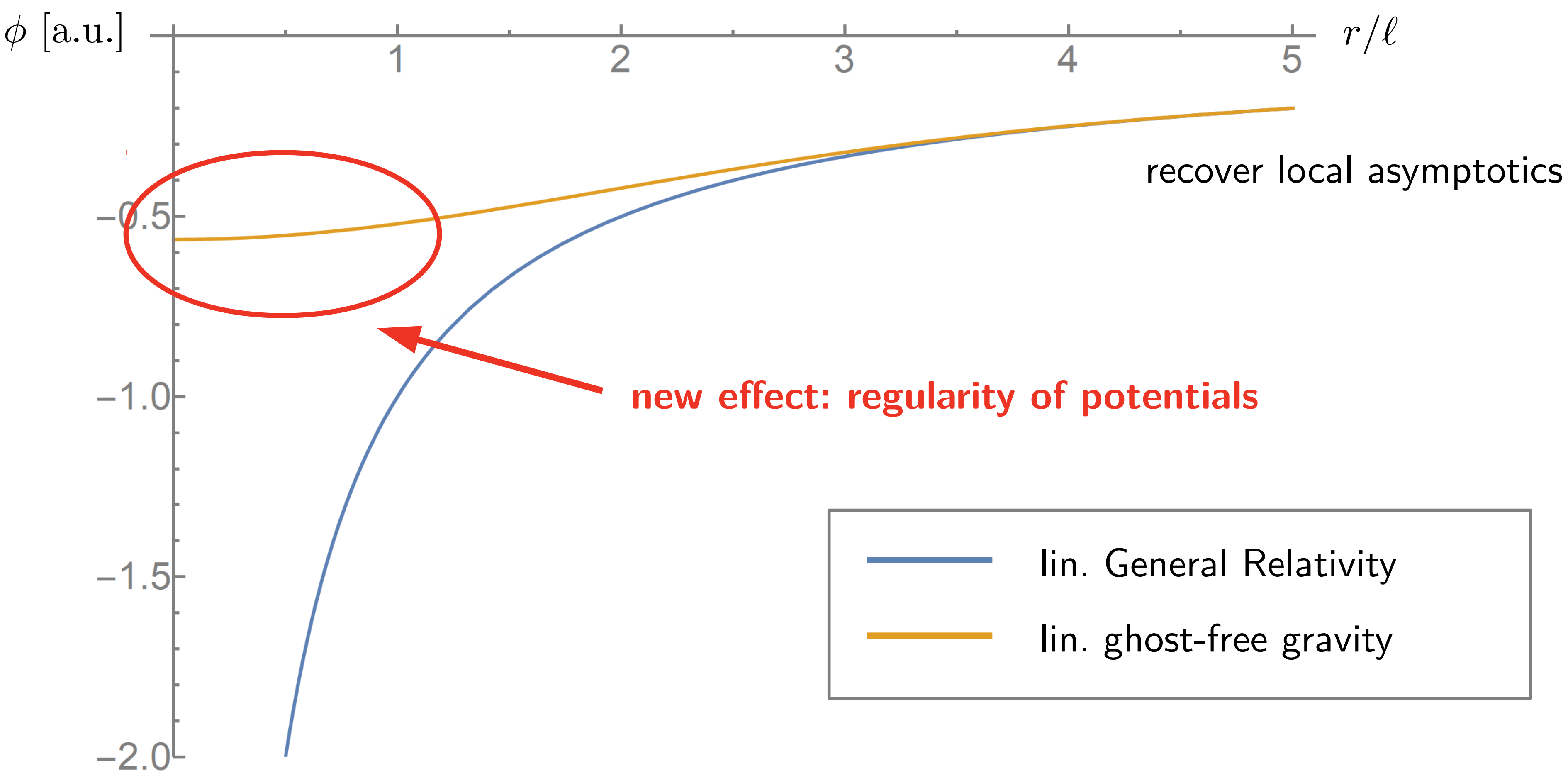
The resulting potential is finite and well-behaved at $r = 0$!

The presence of non-locality leads to a **regularization of the gravitational field** for a wide range of sources (rotating and extended, brane-like objects) and for a large class of ghost-free theories.

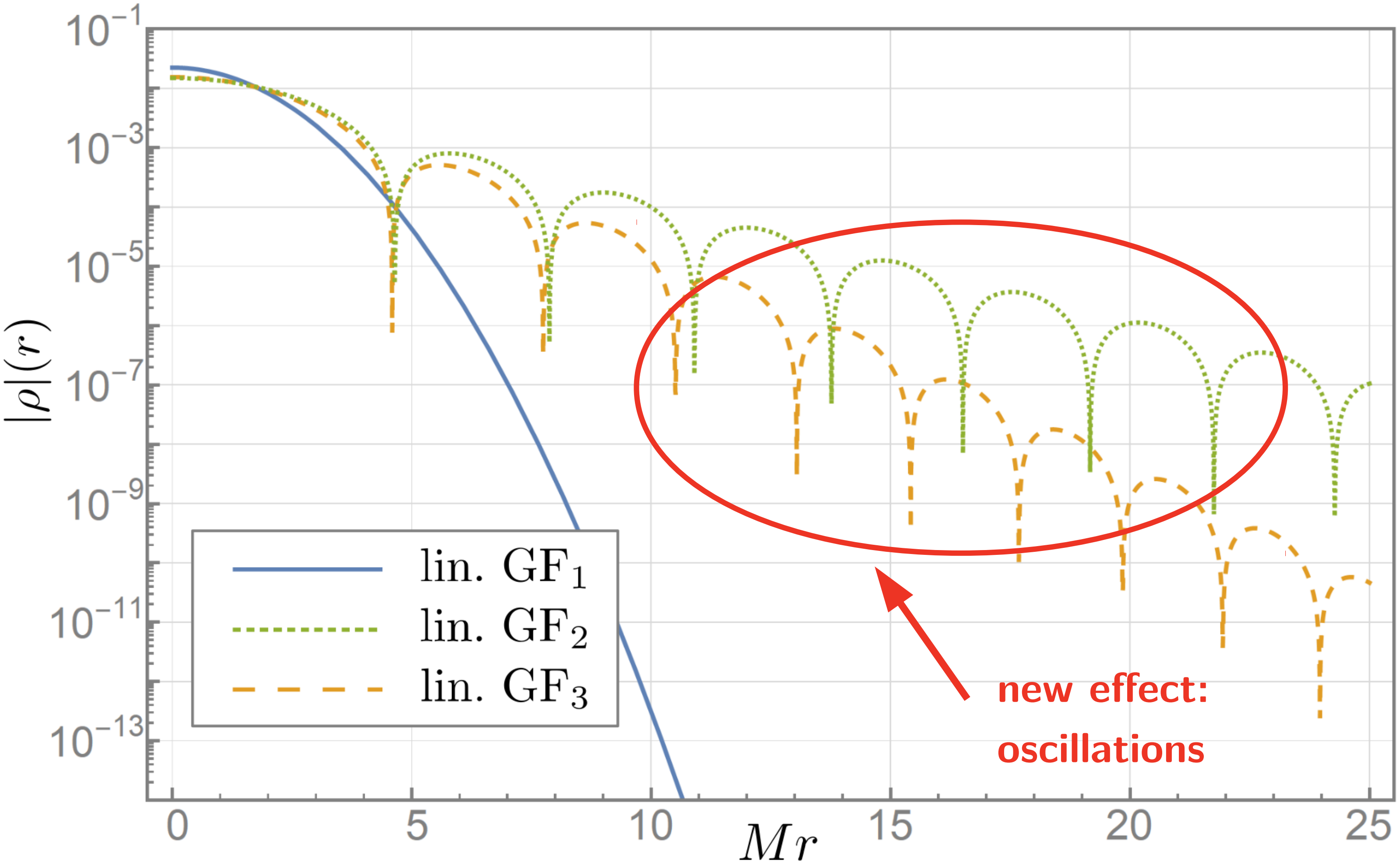
Possible interpretation in terms of effective energy density (here in 1D for simplicity):

$$\rho_{\text{eff}}(r) = \left[e^{-\ell^2\nabla^2}\right]^{-1}\delta(r) = e^{\ell^2\partial_r^2}\int\limits_{-\infty}^{\infty}\frac{\mathrm{d}k}{2\pi}e^{ikr} = \frac{e^{-r^2/(4\ell^2)}}{\sqrt{4\pi}\ell}$$

Weak-field ghost-free gravity: regular point particles and extended objects



Weak-field ghost-free gravity: Friedel oscillations in effective energy density

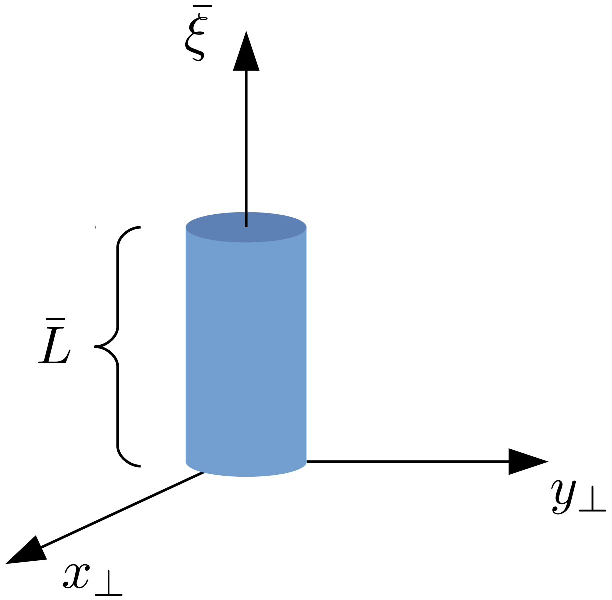


Weak-field ghost-free gravity: ultrarelativistic objects

We developed a **new technique** to study the Penrose limit in wide class of gravitational theories via a special representation of the relevant Green function in terms of the heat kernel $K_d(r|\tau)$:

$$\mathcal{G}_d(r) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}\eta}{a(-\eta\ell^2)\eta} \int\limits_{-\infty}^{\infty} \mathrm{d}\tau K_d(r|\tau) e^{i\eta\tau} \, ,$$

$$K_d(r|\tau) = \frac{1}{(4\pi i\tau)^{d/2}} e^{i\frac{r^2}{4\tau}} \, .$$



The resulting geometries are **regular pp-waves**.

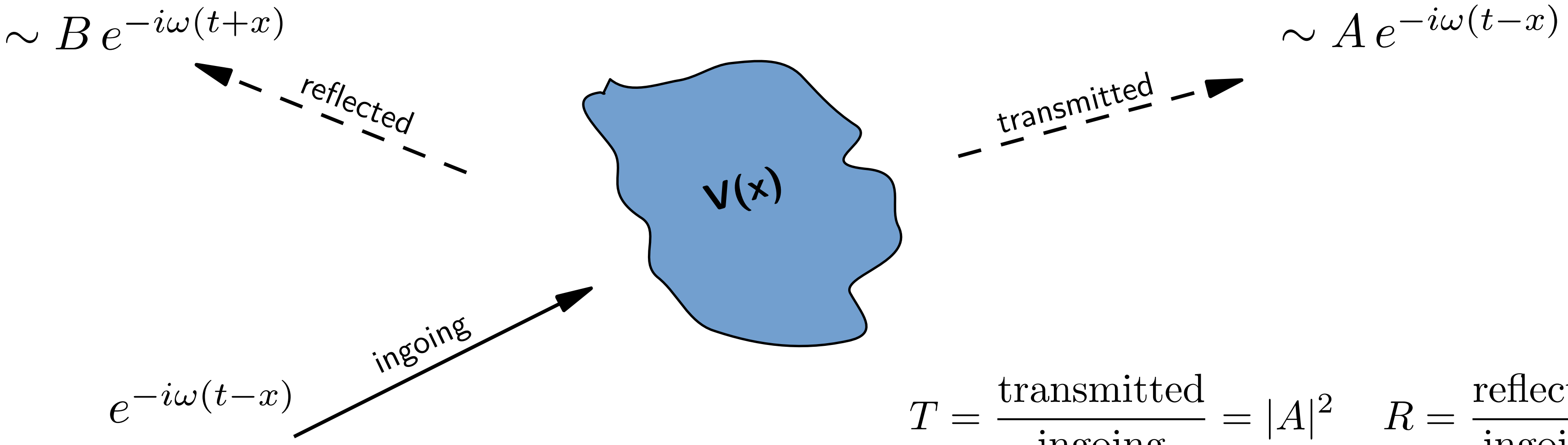
They reduce to Aichlburg–Sexl-type metrics in the limit $\ell \to 0$.

Open question: are these also exact non-linear solutions?

Promising starting point since all scalar invariants vanish due to null gravitational field.

Effects of Non-locality
in Gravity and Quantum Theory

Quantum-mechanical scattering



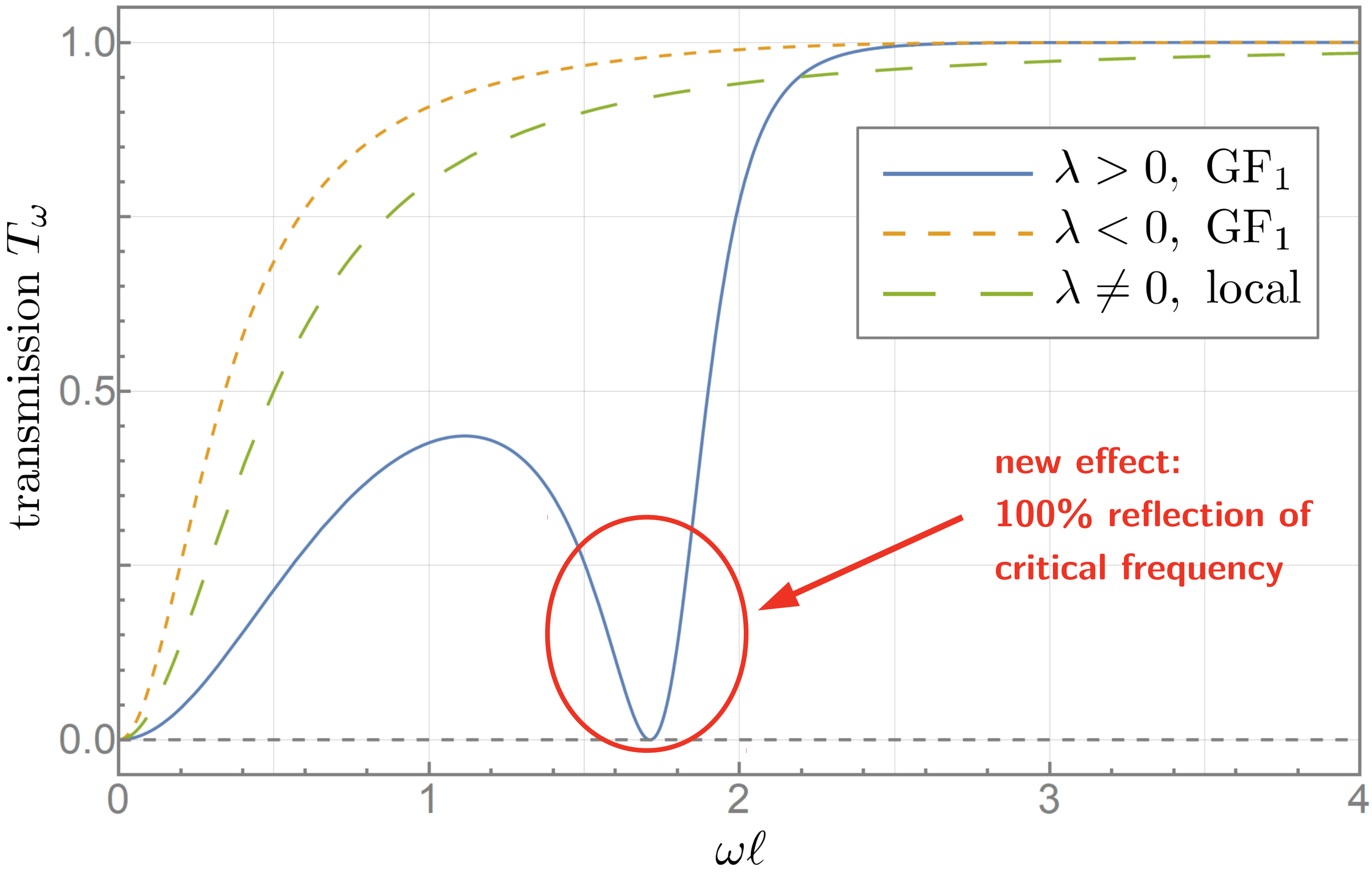
$$T = \frac{\text{transmitted}}{\text{ingoing}} = |A|^2 \qquad R = \frac{\text{reflected}}{\text{ingoing}} = |B|^2$$

Study the non-local scattering of a scalar field on a point-like potential, $f(\Box)\Box\varphi = V(x)\varphi$.

Can find the exact form of $\varphi(x)$ in Fourier space from Lippmann–Schwinger equation:

$$\varphi_\omega(x) = \varphi^0_\omega(x) - \int \mathrm{d}y\, \mathcal{G}^{\mathrm{R}}_d(x-y)V(y)\varphi_\omega(y) = \varphi^0_\omega(x) - \lambda\mathcal{G}^{\mathrm{R}}_d(x)\varphi_\omega(0)$$

Read of transmission coefficients. Unexpected property: **100% reflection of a critical frequency**.



1.0
0.5
0.0
transmission T_ω
0
1
2
3
4
$\omega\ell$
$\lambda > 0$, GF_1
$\lambda < 0$, GF_1
$\lambda \neq 0$, local
new effect:
100% reflection of
critical frequency

Vacuum polarization in quantum field theory

Consider the following expectation value ("renormalized vacuum polarization"):

$$\langle \varphi^2(x) \rangle_{\mathrm{ren}} \equiv \lim_{x \to x'} \Big[\langle \hat{\varphi}(x) \hat{\varphi}(x') \rangle_{V \neq 0} - \langle \hat{\varphi}(x) \hat{\varphi}(x') \rangle_{V=0} \Big] \,.$$

Decompose into temporal Fourier modes and formally write

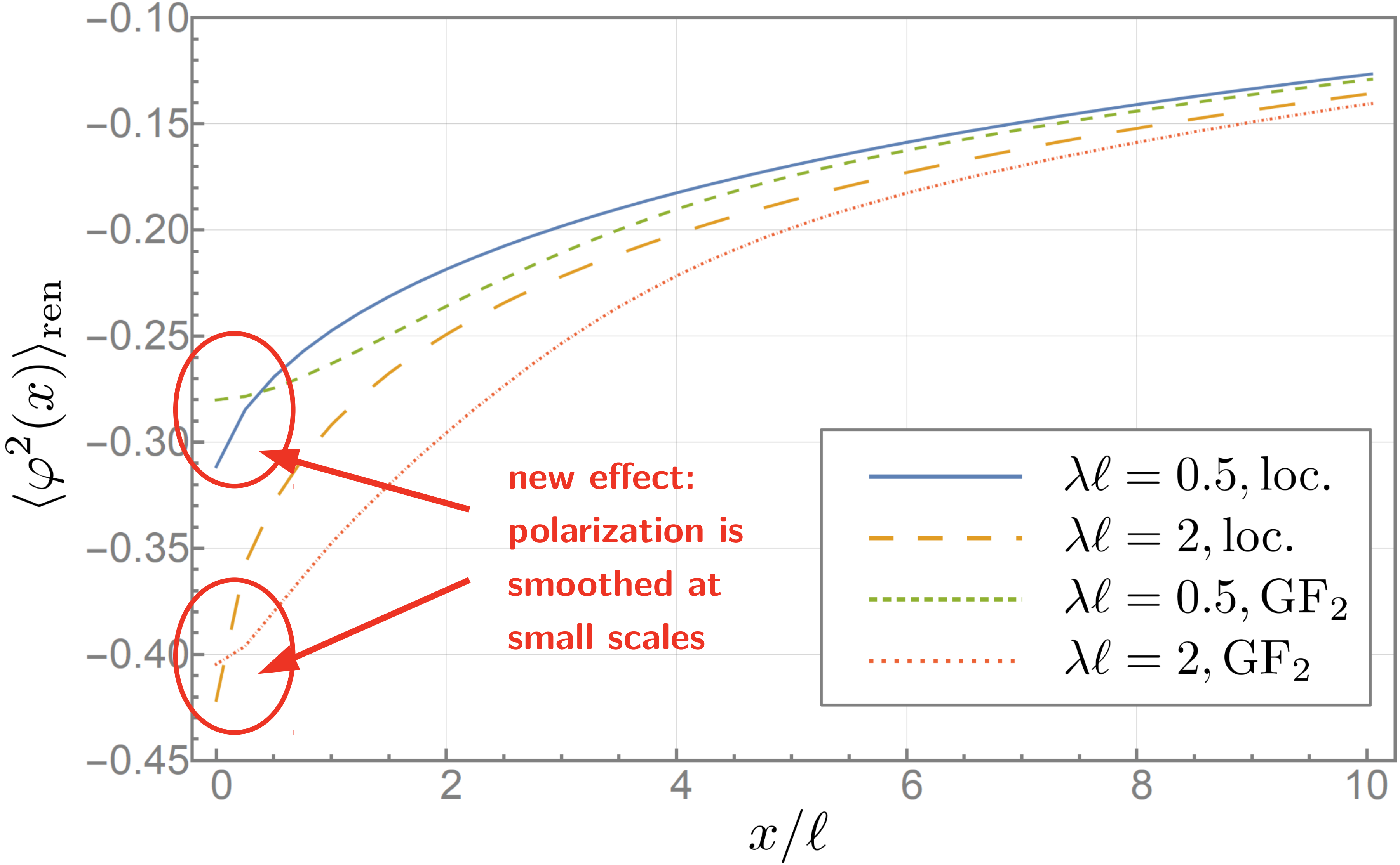
$$\hat{\varphi}_\omega(x) = \hat{\varphi}^0_\omega(x) + \Lambda_\omega(x) \hat{\varphi}^0_\omega(0) \,.$$

Difference to quantum mechanics: integrate over temporal Fourier modes (technically difficult).

Can express all expectation values in terms of the free Hadamard function $f(\Box)\Box G^{(1)}(x', x) = 0\,.$

This procedure is unique and one recovers the results of local QFT in the limit $\ell \to 0$.

Can be extended to finite temperature in accordance with fluctuation-dissipation theorem.



−0.10
−0.15
−0.20
−0.25
−0.30
−0.35
−0.40
−0.45
0
2
4
6
8
10
$\langle \varphi^2(x) \rangle_{\mathrm{ren}}$
x/ℓ
new effect:
polarization is
smoothed at
small scales
$\lambda\ell = 0.5, \mathrm{loc.}$
$\lambda\ell = 2, \mathrm{loc.}$
$\lambda\ell = 0.5, \mathrm{GF}_2$
$\lambda\ell = 2, \mathrm{GF}_2$

Effects of Non-locality
in Gravity and Quantum Theory

Two-dimensional black hole, non-locality, and Hawking radiation [Ch. 7]

Employ quantum field theory in curved spacetime to study interplay of non-locality and black holes.

Take as a starting point a non-local modification of the Polyakov effective action:

$$W_{\mathrm{GF}}^{\mathrm{Pol}}[g_{\mu\nu}] = -\frac{1}{96\pi}\int \mathrm{d}^2x\,\sqrt{-g}R\frac{e^{-(-\ell^2\Box)^N}}{\Box}R$$

Developed integral representation of $e^{-\ell^2\Box}R$ to extract $\langle T_{\mu\nu}\rangle$ for 2D stationary geometry.

Hawking temperature $T = f'(r_g)/(4\pi)$ is **insensitive to non-locality**.
Asymptotic Hawking flux **remains unchanged**, $T^t{}_r \sim (k_B T)^2$.

However, **entropy corrections due to non-locality are non-trivial** and imply back-reaction.
An application to a concrete two-dimensional black hole confirms these general results.

Summary and Conclusions

Original goal: complete understanding of non-linear non-locality in gravity and quantum theory. We studied the linear case which gave rise to **several unexpected and non-trivial effects** that improved our understanding of non-locality in both the classical and the quantum regime:

- Developed general tools applicable to a wide range of non-local ghost-free theories.
- Obtained a rather complete description of the properties of this class of theories.
- These results can serve as a stepping stone towards non-linear studies.

Open questions:

- Does the regularity of gravitational potentials imply finite non-linear solutions?
- Can the non-locality affect the information loss paradox?

In the future I hope to address open problems that involve non-locality in connection with black hole physics and cosmology. **Thank you for your attention.**