Ultrarelativistic objects in non-local infinite-derivative gravity

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Particle Physics Seminar
College of William & Mary, Williamsburg, VA, USA
Tuesday, June 9, 2020, 3:00pm

Based on...

Ultrarelativistic spinning objects (gyratons) in non-local ghost-free gravity

Jens Boos,[1,*] Jose Pinedo Soto,[1,†] and Valeri P. Frolov[1,‡]

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(Dated: April 17, 2020)

We study the gravitational field of ultrarelativistic spinning objects (gyratons) in a modified gravity theory with higher derivatives. In particular, we focus on a special class of such theories with an infinite number of derivatives known as "ghost-free gravity" that include a non-local form factor such as $\exp(-\Box\ell^2)$, where ℓ is the scale of non-locality. First, we obtain solutions of the linearized ghost-free equations for stationary spinning objects. To obtain gyraton solutions we boost these metrics and take their Penrose limit. This approach allows us to perform calculations for any number of spacetime dimensions. All solutions are regular at the gyraton axis. In four dimensions, when the scale non-locality ℓ tends to zero, the obtained gyraton solutions correctly reproduce the Aichelburg–Sexl metric and its generalization to spinning sources found earlier by Bonnor. We also study the properties of the obtained four-dimensional and higher-dimensional ghost-free gyraton metrics and briefly discuss their possible applications.

arXiv: 2004.07420 [gr-qc], to appear in Phys. Rev. D.

More work on ghost-free gravity with Valeri Frolov and Andrei Zelnikov.

Overview

infinitely many derivatives

infinite-derivative gravity

ultrarelativistic objects

PART I

INFINITELY MANY DERIVATIVES

motivation ▪ ghost-free propagators ▪ initial value problem

Infinite derivatives cure short-distance singularities (1/3)

Original motivation stems from Pauli–Villars regularization in quantum field theory. Let us give an example in the context of the classical theory of Newtonian gravity. (Later: General Relativity!)

The gravitational potential of a point-particle is singular at the origin ($\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$).

$$\nabla^2 \phi = 4\pi G m \delta(\boldsymbol{r}) \quad \longrightarrow \quad \phi(r) = \frac{-Gm}{r}$$

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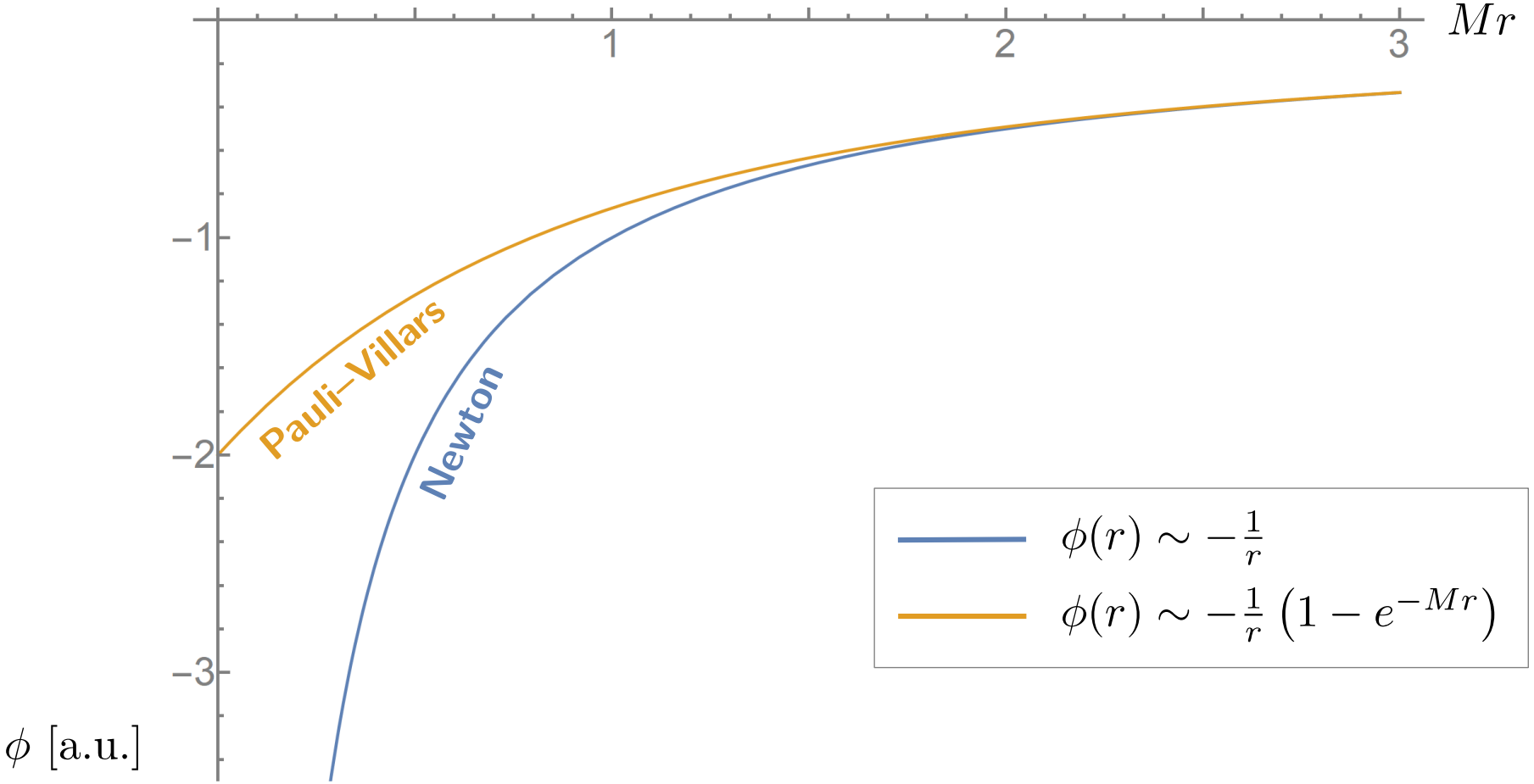
The pathological behavior at $r = 0$ can be cured by introducing a **heavy-mass modification**:

$$\nabla^2 (1 - \nabla^2/M^2)\phi = 4\pi G m \delta(\boldsymbol{r}) \quad \longrightarrow \quad \phi(r) = \frac{-Gm}{r}\left(1 - e^{-Mr}\right)$$

This is called Pauli–Villars regularization, and we assume that $M \gg m$ (short distance modification). For large distances the potential is Newtonian, but for short distance scales it is regularized.


Mr
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Pauli–Villars
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The Green function of the Pauli–Villars regularized theory has the following structure:

$$G(\boldsymbol{x}', \boldsymbol{x}) \sim \frac{1}{\nabla^2 \left(1 - \nabla^2/M^2\right)} = \frac{1}{\nabla^2} - \frac{1}{\nabla^2 - M^2}$$

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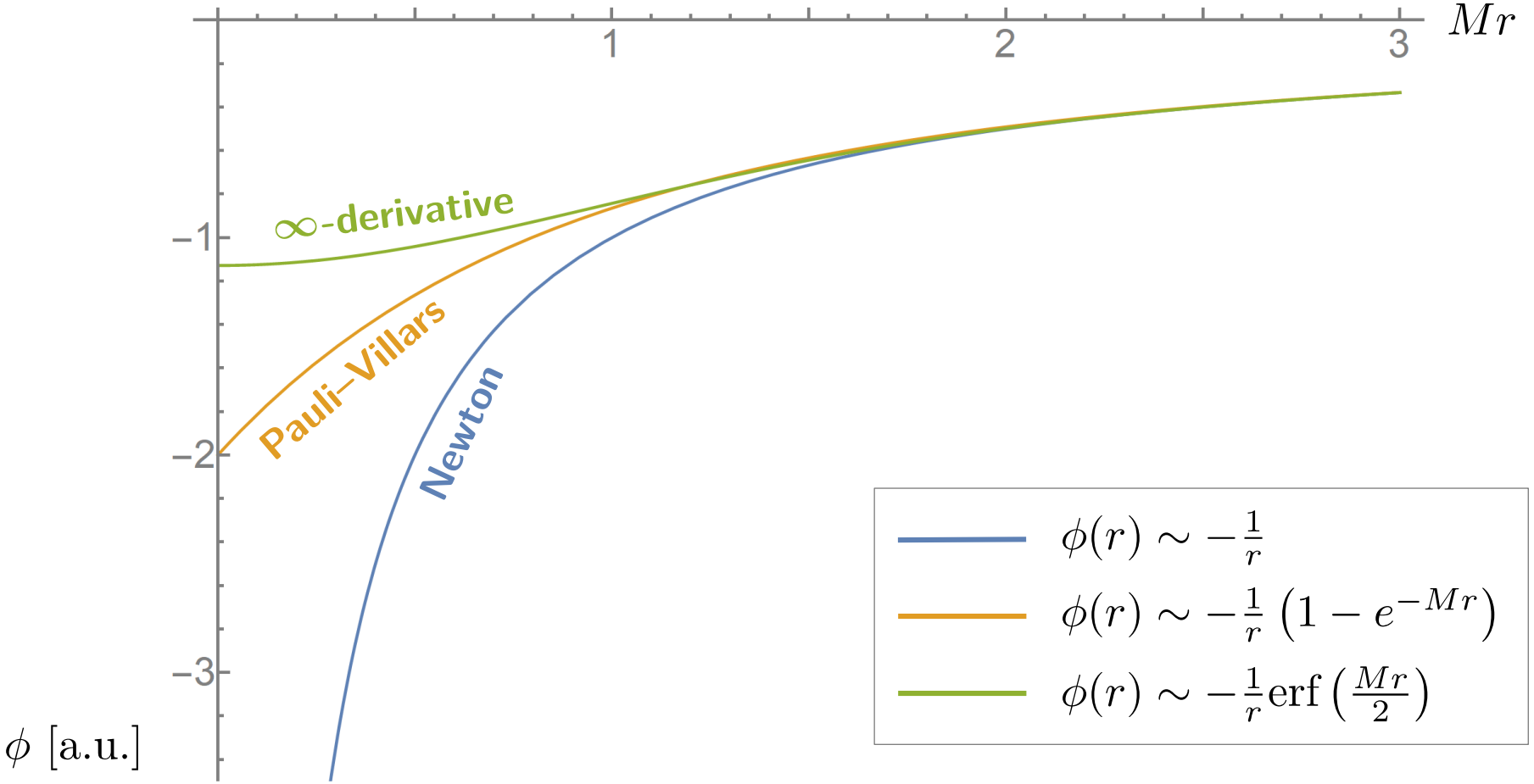
Can we find a regularization that does not introduce any new degrees of freedom?

Infinite derivatives cure short-distance singularities (3/3)

How about using an **infinite** amount of derivatives?

$$\nabla^2 e^{-\nabla^2/M^2}\phi = 4\pi G m \delta(\boldsymbol{r}) \quad \longrightarrow \quad \phi(r) = \frac{-Gm}{r}\mathrm{erf}\left(\frac{Mr}{2}\right)$$

We will derive this in more detail later.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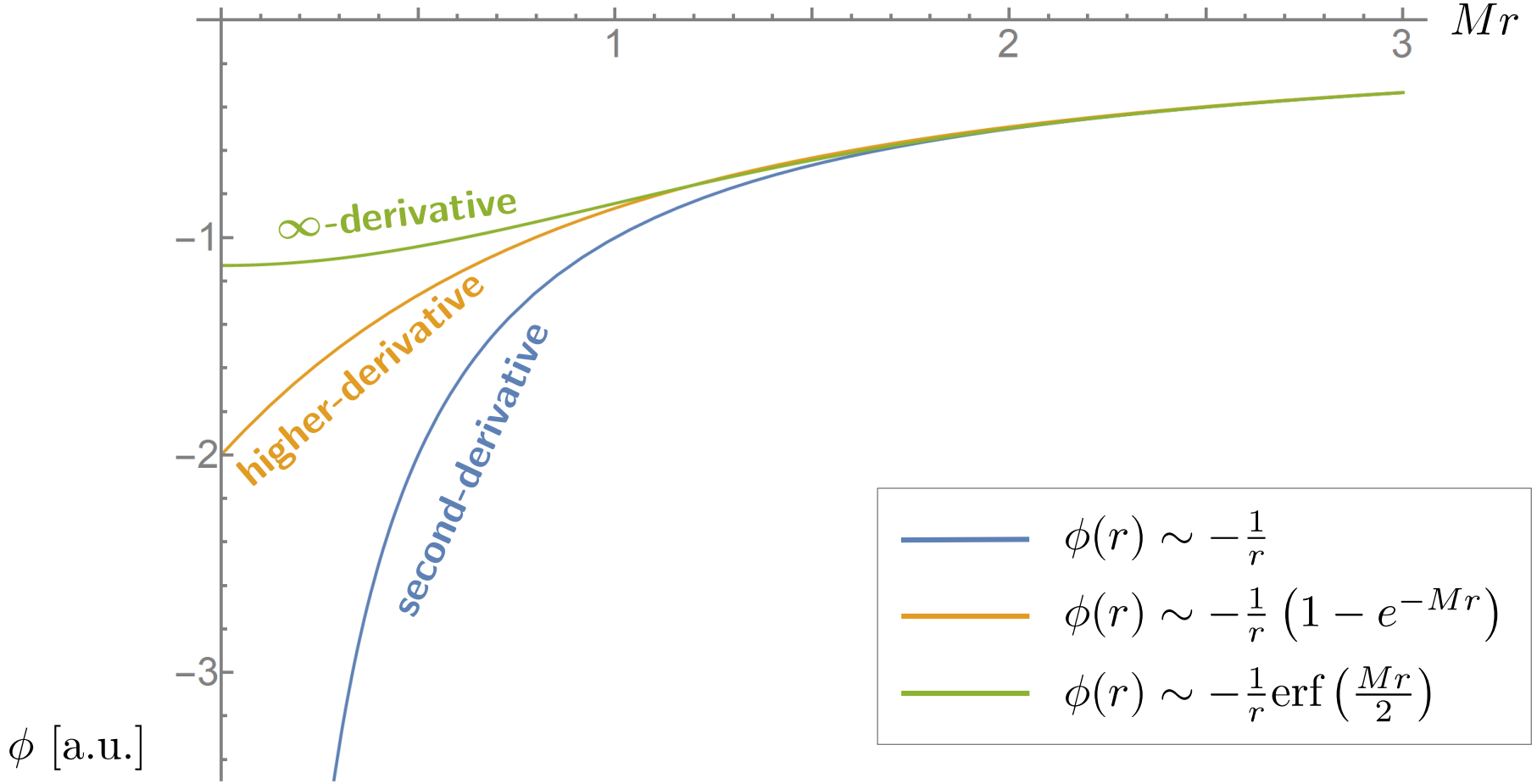
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$$f(\nabla^2) = f(\triangle) = \exp(-\triangle/M^2) \equiv \sum_{n=0}^{\infty} \frac{(-1)^n \triangle^n}{M^{2n} n!}$$

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The object $f(\triangle)$ is often called a **form factor**. It has the following important properties:

- $f > 0$ "ghost-free condition"
- $f(0) = 1$ "on-shell" condition

Let's understand the role of these conditions better!

The "ghost-free" condition $f > 0$ (1/2)

For a general form factor $f(\triangle)$ and current $j(\boldsymbol{r})$ the Poisson equation and Green function are

$$\triangle f(\triangle)\phi = j(\boldsymbol{r})\,, \qquad G(\boldsymbol{x'}, \boldsymbol{x}) \sim \frac{1}{\triangle f(\triangle)}\,.$$

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Here, $j_{\text{eff}}(\boldsymbol{r})$ is a coarse-grained current. Two interpretations of ghost-free infinite derivatives:

- Infinite-derivative field equations with a given source.
- Second-order field equations with an effective source.

Let us calculate the effective current for a specific example!

The "ghost-free" condition $f > 0$ (2/2)

Calculate for $f(\triangle) = e^{-\triangle/M^2}$ and $j(\boldsymbol{r}) = \delta(\boldsymbol{r})$ in one spatial dimension ($\triangle = \partial_r^2$):

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This result is just a Gaussian with a width $\ell = M^{-1}$, justifying the interpretation of **infinite-derivative theories as non-local theories**, where ℓ is the **scale of non-locality**.

Non-locality = mathematical sandpaper

The "on-shell" condition $f(0) = 1$

One can find the following representation for the static non-local d-dimensional Green function:

$$\triangle f(\triangle) G_d(\boldsymbol{x}', \boldsymbol{x}) = -\delta^{(d)}(\boldsymbol{x}' - \boldsymbol{x}) \,,$$

$$G_d(\boldsymbol{x}', \boldsymbol{x}) = \frac{1}{(2\pi)^{d/2} r^{d-2}} \int\limits_0^\infty \mathrm{d}\zeta \zeta^{\frac{d-4}{2}} \frac{1}{f(-\zeta^2 \ell^2 / r^2)} J_{\frac{d}{2}-1}(\zeta) \,, \quad r = |\boldsymbol{x}' - \boldsymbol{x}| \,.$$

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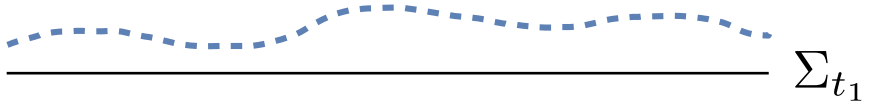
Origin of the term "on-shell": $\triangle \chi = 0 \quad \Rightarrow \quad f(\triangle)\chi = f(0)\chi = \chi$

Interesting fact: solutions of homogeneous equations are the same in local and ghost-free theories!

Initial value problem in infinite-derivative ghost-free theories



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- specify φ and $\partial_t \varphi$ on Σ_{t_1}
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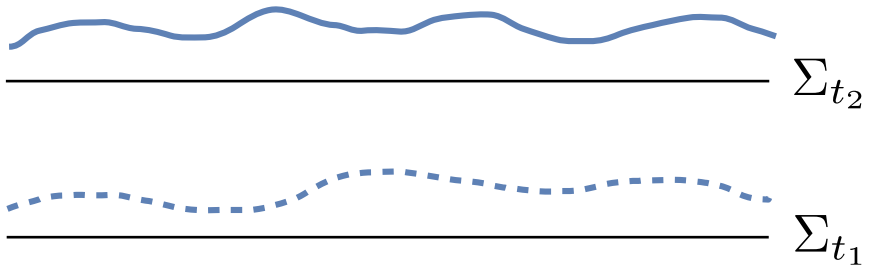

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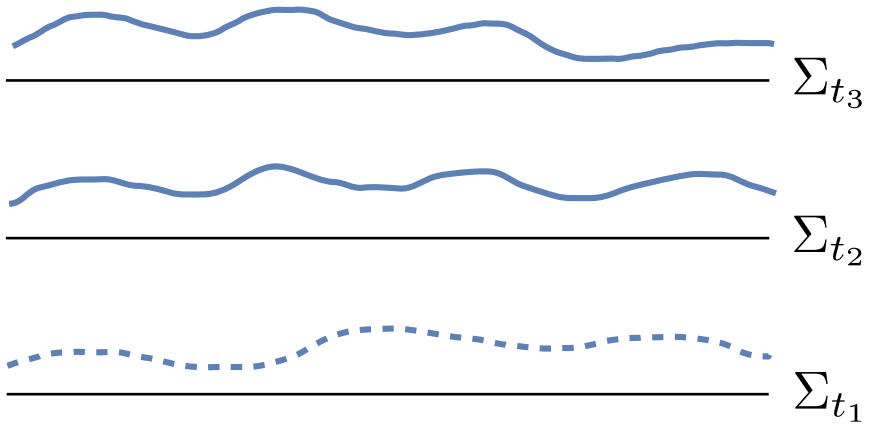

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Initial value problem in infinite-derivative ghost-free theories

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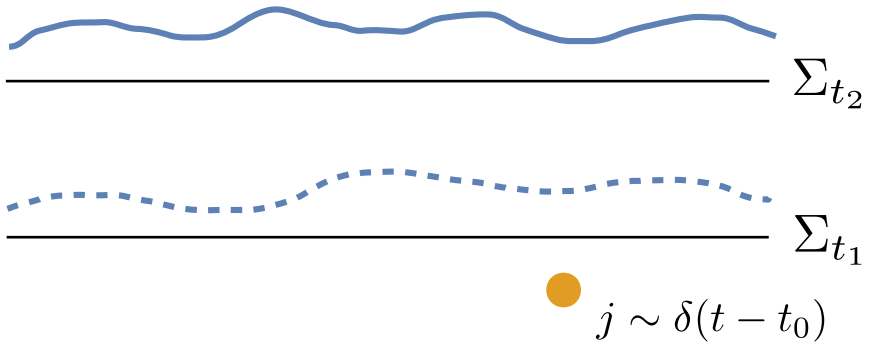

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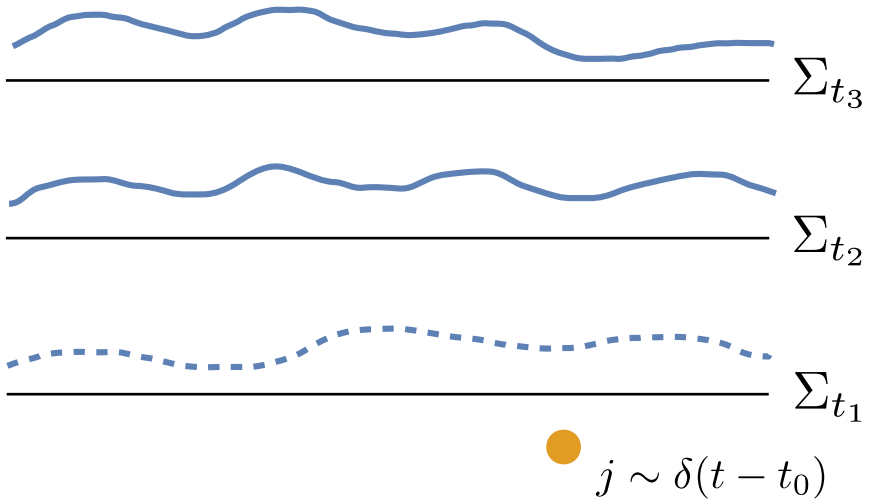

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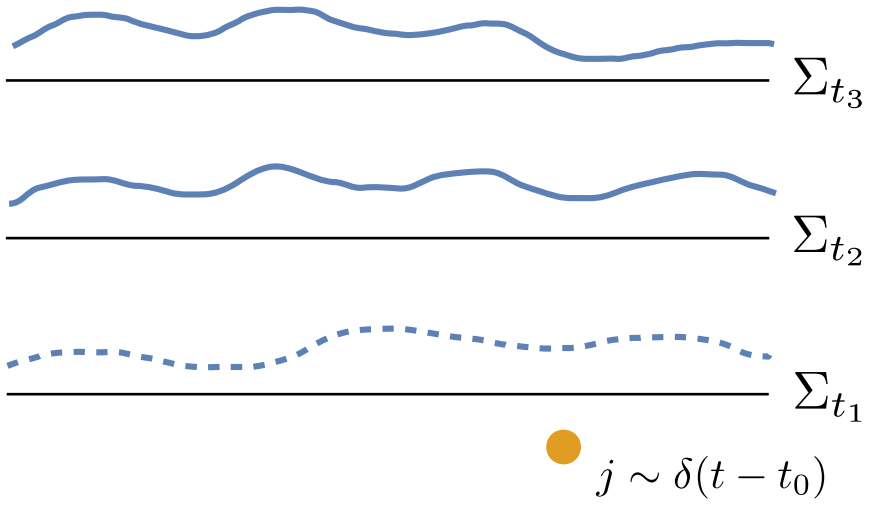

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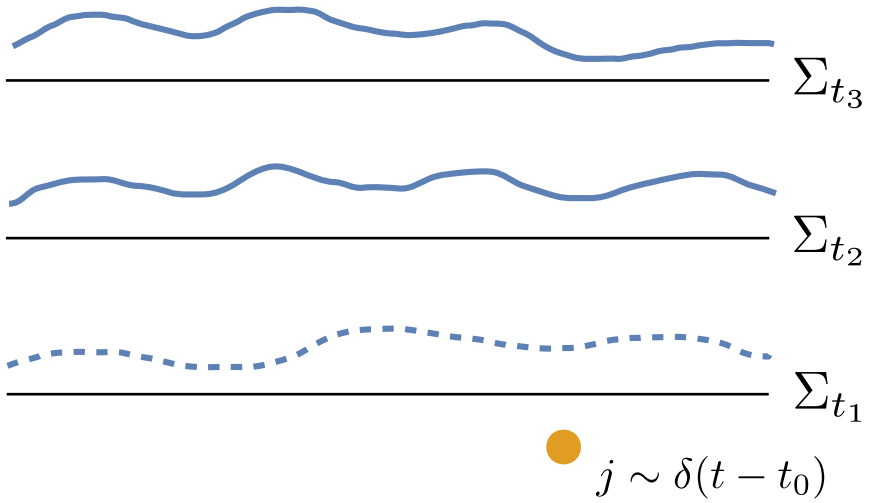

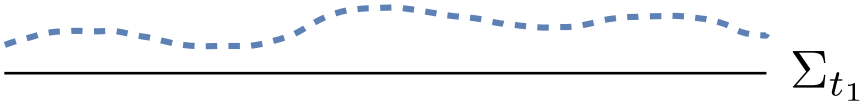

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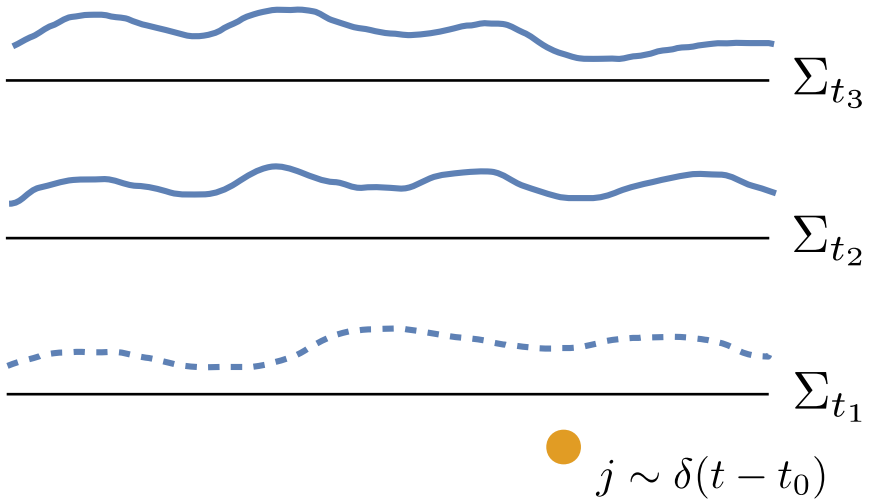

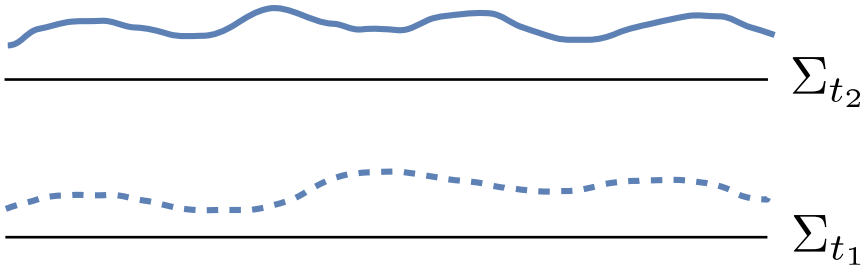

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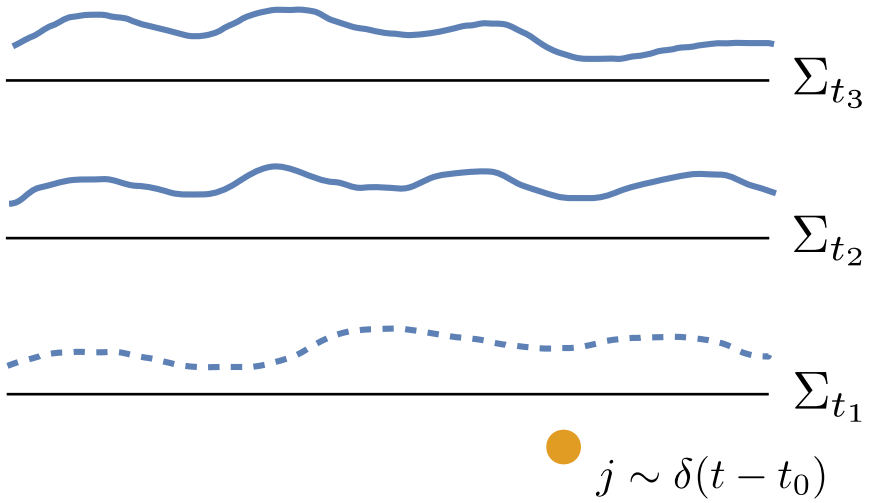
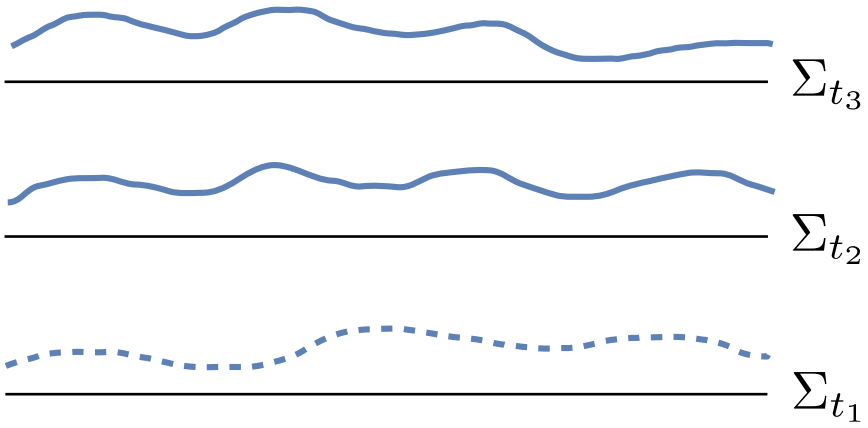
Initial value problem in infinite-derivative ghost-free theories





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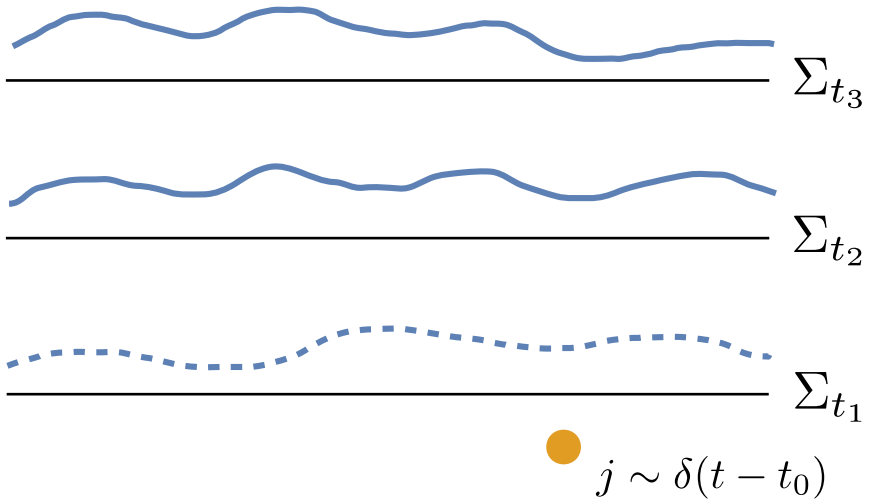

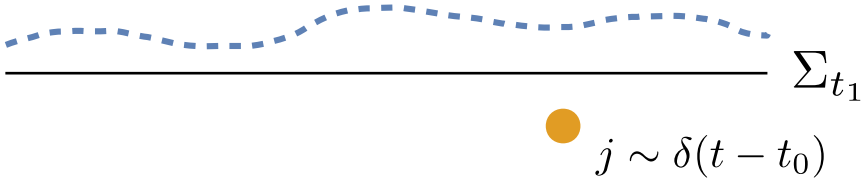

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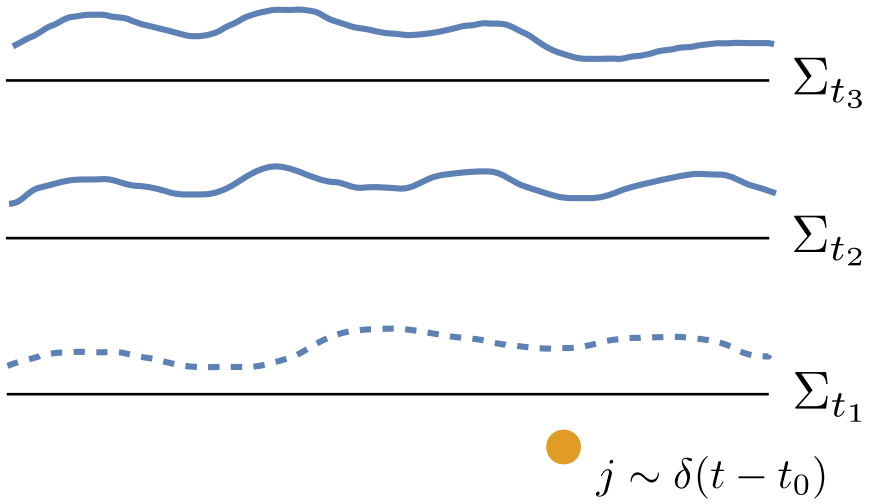

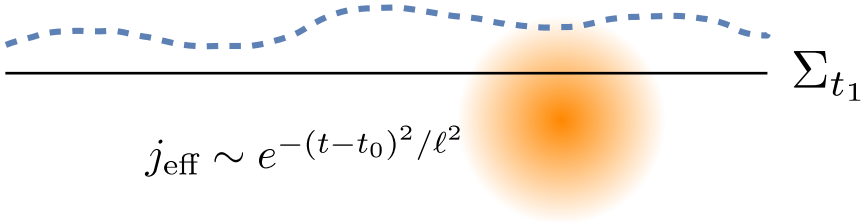

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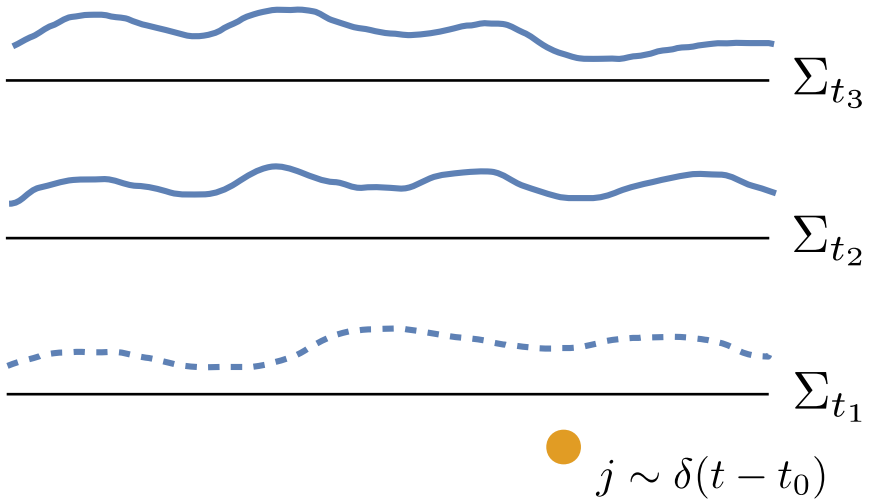

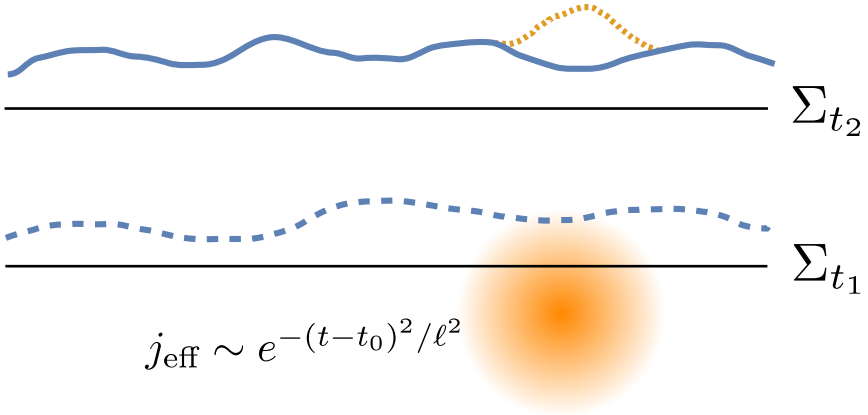

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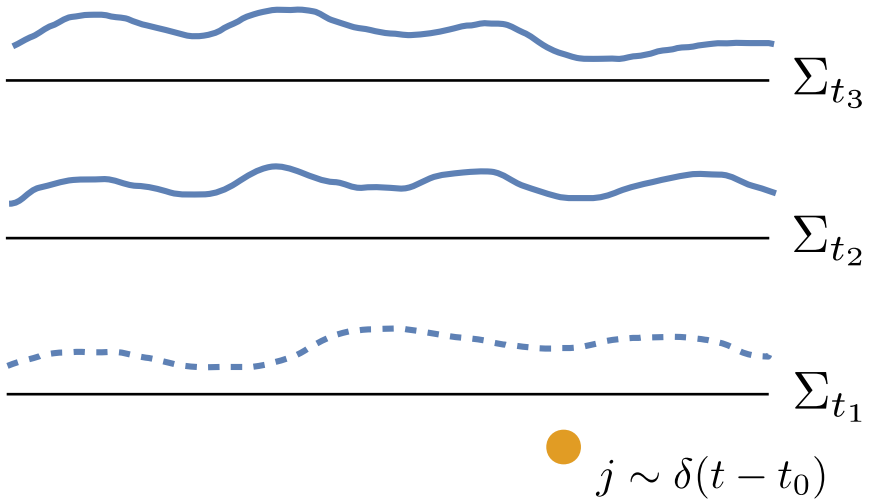

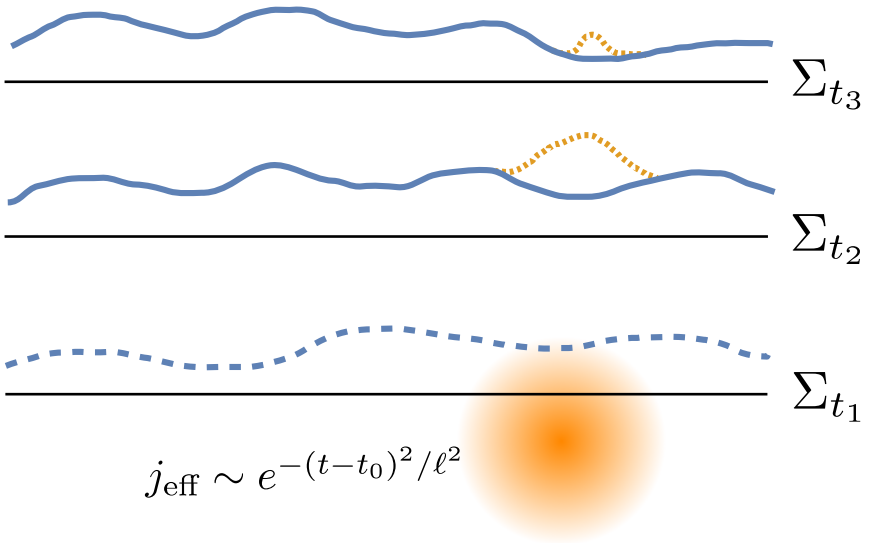

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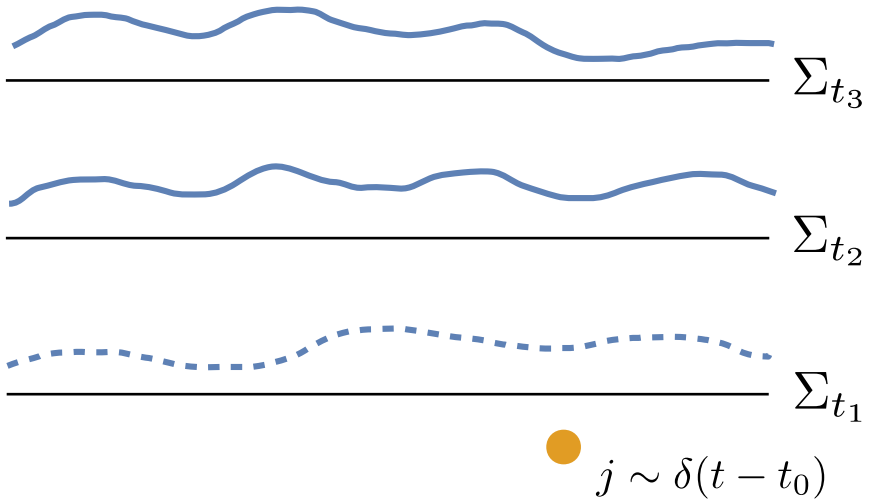

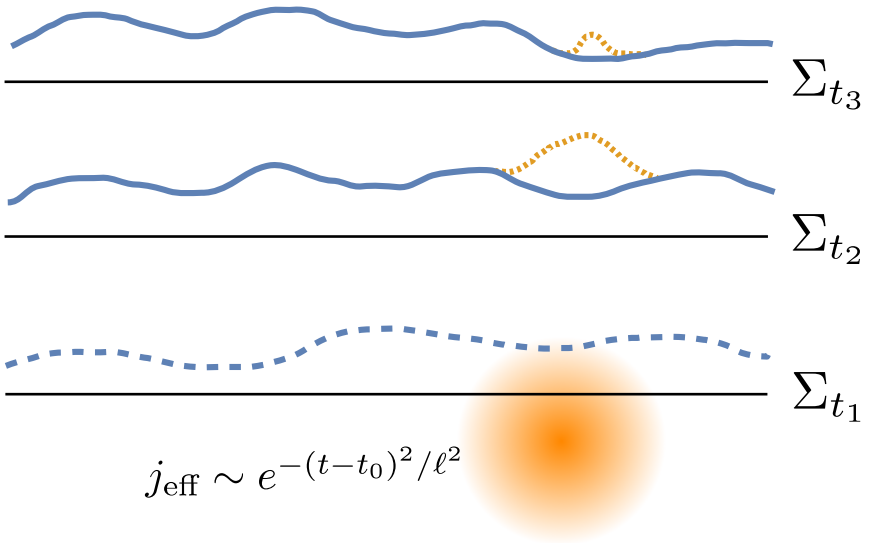

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Formal treatment (Barnaby et al): amount of initial data corresponds to poles in propagator.

PART II

INFINITE-DERIVATIVE GRAVITY

singularities ▪ linearized field equations ▪ static & regular solutions

General Relativity, linearized and exact (1/2)

Consider a small perturbation around Minkowski space, $g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}$ with $\epsilon \ll 1$.

$$
\begin{aligned}
S[h_{\mu\nu}] &= \frac{1}{2\kappa} \int \sqrt{-g} \mathrm{d}^D x R \\
&= \frac{1}{2\kappa} \int \mathrm{d}^D x \Big(\frac{1}{2} h^{\mu\nu} \Box h_{\mu\nu} - h^{\mu\nu} \partial_\mu \partial_\alpha h^\alpha{}_\nu + h^{\mu\nu} \partial_\mu \partial_\nu h - \frac{1}{2} h \Box h \Big) + \mathcal{O}(\epsilon^3) \,.
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Field equations (harmonic gauge $\partial^\mu \hat{h}_{\mu\nu} = 0$ with $\hat{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$):

$$\Box \hat{h}_{\mu\nu} = -2\kappa T_{\mu\nu}$$

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What is the gravitational field for a particle of mass m at rest?

$$T_{\mu\nu} = \delta^t_\mu \delta^t_\nu \, m \delta^{(d)}(\boldsymbol{x})$$

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Let us compare the linearized solution to the known exact solution (Schwarzschild black hole).

General Relativity, linearized and exact (2/2)

The Schwarzschild solution (Droste 1916; Schwarzschild 1916) can be written as

$$g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = -\left(1-\frac{2Gm}{r}\right)\mathrm{d}t^2 + \left(1-\frac{2Gm}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\,.$$

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Linearized solution:

$$(\eta_{\mu\nu}+h_{\mu\nu})\,\mathrm{d}x^\mu\mathrm{d}x^\nu = -\left(1-\frac{2Gm}{r}\right)\mathrm{d}t^2 + \left(1+\frac{2Gm}{r}\right)\mathrm{d}\vec{x}^{\,2}$$

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The Schwarzschild solution has a **curvature singularity** at $r=0$ since $C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} \sim \frac{m^2}{r^6}$.
This singularity also exists at the linearized level, and even in the Newtonian limit.

Question: What is the linear solution of infinite-derivative gravity, and what can we learn from it?

Linearized infinite-derivative gravity (1/2)

Consider a small perturbation around Minkowski space, $g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}$ with $\epsilon \ll 1$.

$$S[h_{\mu\nu}] = \frac{1}{2\kappa} \int \sqrt{-g} \mathrm{d}^D x \left(R + \frac{1}{2} R_{\mu\nu\rho\sigma} \mathcal{O}^{\mu\nu\rho\sigma}_{\alpha\beta\gamma\delta}(\nabla, \Box) R^{\alpha\beta\gamma\delta} \right)$$

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$$- \frac{1}{2}h\, \boldsymbol{c}(\Box)\Box h + \frac{1}{2}h^{\mu\nu}\, \frac{\boldsymbol{a}(\Box) - \boldsymbol{c}(\Box)}{\Box}\partial_\mu\partial_\nu\partial_\alpha\partial_\beta\, h^{\alpha\beta}\Big) + \mathcal{O}(\epsilon^3)\,.$$

Let us set the two **form factors** equal, $\boldsymbol{a}(\Box) = \boldsymbol{c}(\Box)$, and then the linear field equations are

$$a(\Box)\Box\hat{h}_{\mu\nu} = -2\kappa T_{\mu\nu}\,, \quad \hat{h}_{\mu\nu} = \hat{h}^0_{\mu\nu} + 2\kappa\int \mathrm{d}^D y\, G(\boldsymbol{x}, \boldsymbol{y})T_{\mu\nu}(\boldsymbol{y})\,.$$

Given $T_{\mu\nu}$, this can be solved with the Green function method.

Linearized infinite-derivative gravity (2/2)

Let us focus on a simple case when $a(\Box) = e^{-\ell^2 \Box} = e^{-\ell^2 \triangle}$. We could have guessed the solution:

$$(\eta_{\mu\nu} + h_{\mu\nu})\, \mathrm{d}x^\mu \mathrm{d}x^\nu = -\,(1+\phi)\, \mathrm{d}t^2 + (1-\phi)\, \mathrm{d}\vec{x}^{\,2}\,, \quad \phi(r) = -\frac{2Gm}{r} \mathrm{erf}\left(\frac{r}{2\ell}\right)\,.$$

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This metric and all its curvature invariants are regular at $r = 0$. But so what?

- An exact solution of the non-linear infinite-derivative gravity field equations for a point particle has not been found yet due to the complicated nature of the equations.
- From the local case we know that the principle of general covariance *does not* remove the singularity of the Newtonian potential.
- Can show that $1/r$-metric does not solve Ricci flat field equations.
- Linear problem in GR is not self-consistent, but it is self-consistent in non-local theory.

We need to find out if we can trust the linear solution. Idea: ultrarelativistic objects!

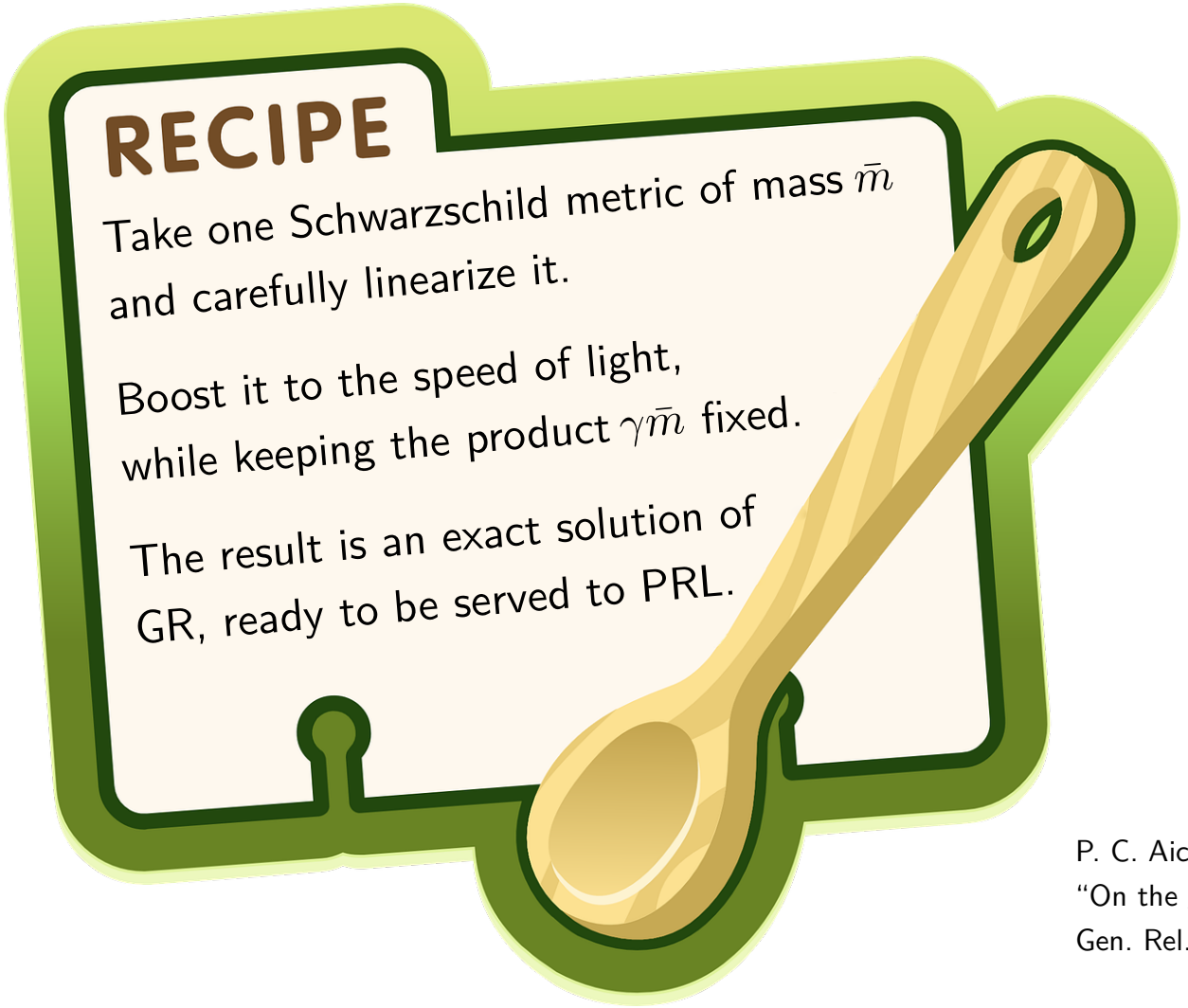
PART III

ULTRARELATIVISTIC OBJECTS

Aichelburg–Sexl metric ▪ Penrose limit ▪ mini black hole formation

Ultrarelativistic objects

The gravitational field of light and ultrarelativistic objects can have rather surprising properties. In 1971, Aichelburg and Sexl demonstrated such an interesting property.



P. C. Aichelburg and R. U. Sexl,
"On the gravitational field of a massless particle,"
Gen. Rel. Grav. **2** (1971), 303–312.

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- Take the linearized Schwarzschild metric (with mass parameter $\bar{m}$) as a "seed" metric.
- Then, perform a boost to a velocity β.
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The intuitive explanation is simple (but not accurate):

- The particle becomes asymptotically null, and the curvature scales as $R \sim \dfrac{\bar{m}}{r^3} \sim \dfrac{m}{\gamma r^3} \to 0$.
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- But: this only works in four spacetime dimensions.

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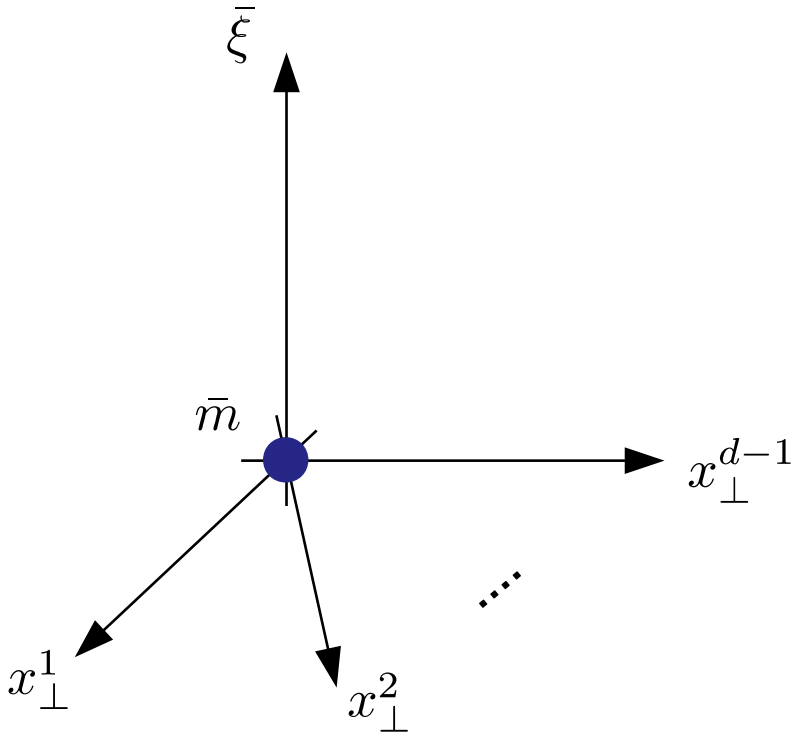
Now: follow the same recipe for infinite-derivative gravity!

Step 1: Linearized metric, pre-boost



Weak-field metric of a point particle:

$$\mathrm{d}\bar{s}^2 = -\left(1-\bar{\phi}\right)\mathrm{d}\bar{t}^2 + \left(1+\bar{\phi}\right)\left(\mathrm{d}\boldsymbol{x}_\perp^2 + \mathrm{d}\bar{\xi}^2\right) ,$$

$$\bar{\phi}(\bar{r}) = \kappa\bar{m}\frac{d-2}{d-1}G_d(\bar{r}) , \quad \bar{r}^2 = \boldsymbol{x}_\perp^2 + \bar{\xi}^2 .$$

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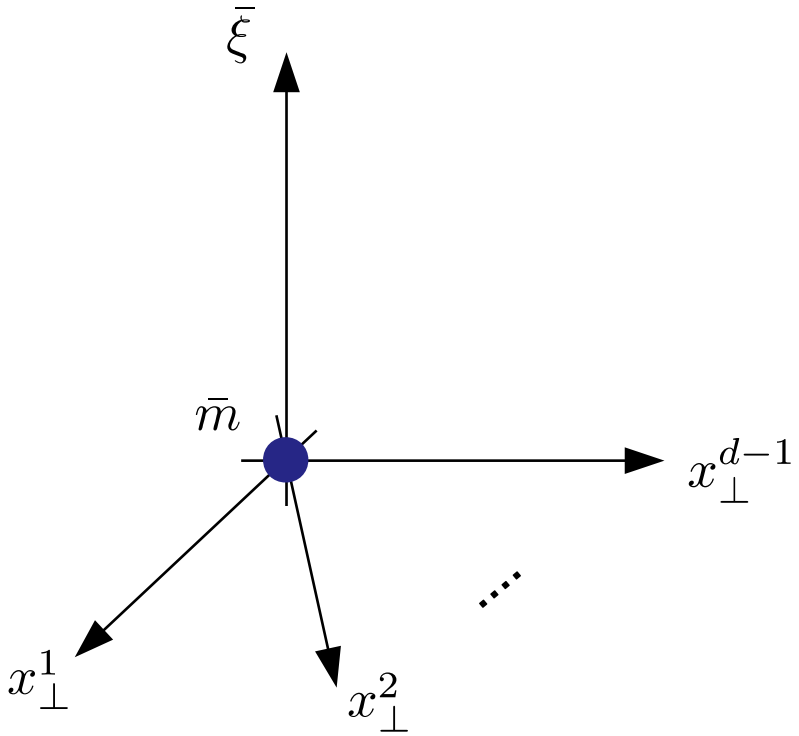

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Now, introduce boosted coordinates $t = \gamma(\bar{t} + \beta\bar{\xi})$ and $\xi = \gamma(\bar{\xi} + \beta\bar{t})$.
Let us also introduce the null coordinates $u = \frac{1}{\sqrt{2}}(t-\xi)$ and $v = \frac{1}{\sqrt{2}}(t+\xi)$.

Step 2: Penrose limit

Boosted metric in the limit $\beta \to 1$ is a pp-wave:

$$\mathrm{d}s^2 = -2\mathrm{d}u\mathrm{d}v + \phi\mathrm{d}u^2 + \mathrm{d}\boldsymbol{x}_\perp^2$$

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$$\phi = \lim_{\gamma \to \infty} 2\gamma^2 \kappa \bar{m}\, G_d\left(\sqrt{2\gamma^2 u^2 + \boldsymbol{x}_\perp^2}\right) = \sqrt{2}\kappa m \delta(u) G_{d-1}(r_\perp)\,, \quad r_\perp = |\boldsymbol{x}_\perp|\,.$$

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There is a lot of physics in this expression:

- This potential describes a **shock wave** that moves along $u = \mathrm{const}$.
- The gravitational potential is given by the $(d-1)$-dimensional Green function $G_{d-1}(r_\perp)$.
- $G_{d-1}(r_\perp)$ is **singular** at $r_\perp = 0$ in linearized General Relativity.
- $G_{d-1}(r_\perp)$ is **finite** at $r_\perp = 0$ in linearized ghost-free gravity.

Gyraton metrics in four spacetime dimensions

Linearized General Relativity:

$$\mathrm{d}s^2 = -2\mathrm{d}u\mathrm{d}v + \phi\mathrm{d}u^2 + \mathrm{d}\rho^2 + \rho^2\mathrm{d}\varphi^2\,, \quad \phi = -\frac{\sqrt{2}\kappa\delta(u)}{2\pi}\log\rho\,, \quad \rho^2 = x^2 + y^2\,.$$

Linearized ghost-free gravity:

$$\mathrm{d}s^2 = -2\mathrm{d}u\mathrm{d}v + \phi\mathrm{d}u^2 + \mathrm{d}\rho^2 + \rho^2\mathrm{d}\varphi^2\,, \quad \phi = -\frac{\sqrt{2}\kappa\delta(u)}{4\pi}\mathrm{Ein}\left(\frac{\rho^2}{4\ell^2}\right)\,.$$

Gyraton metrics in four spacetime dimensions

Linearized General Relativity:

$$\mathrm{d}s^2 = -2\mathrm{d}u\mathrm{d}v + \phi\mathrm{d}u^2 + \mathrm{d}\rho^2 + \rho^2\mathrm{d}\varphi^2\,, \quad \phi = -\frac{\sqrt{2}\kappa\delta(u)}{2\pi}\log\rho\,, \quad \rho^2 = x^2 + y^2\,.$$

Linearized ghost-free gravity:

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One can check that all scalar invariants vanish for these spacetimes ("VSI spacetimes").
This is expected, since they possess a covariantly constant null Killing vector $\boldsymbol{k} = \partial_v$.

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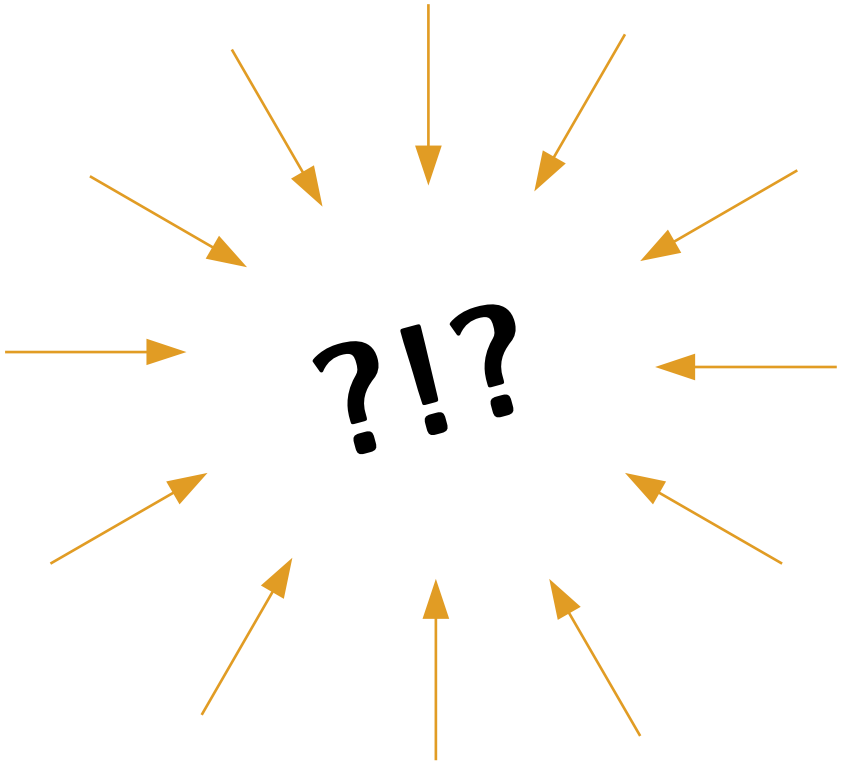
Okay, but what can we learn from this class of metrics?

Collision of ultrarelaticistic objects and black hole formation

This part is based on Frolov et al. Here, we just give the general idea:



- Take a continuum of **gyratons** and let them collide in a small area.
- Average over all fields and obtain the total metric, and check where $g^{rr} = 0$.
- Result: a black hole can only form if the total mass of gyratons exceeds a critical **mass gap**. (But similar results in higher-derivative theories as well.)

PART IV

CONCLUSIONS AND OUTLOOK

congrats ▪ we ▪ made ▪ it

Conclusions and outlook

Infinite-derivative ghost-free physics is very interesting and has several open problems.
We find it insightful to study concrete problems, and demonstrated that:

- The weak-field limit of ghost-free gravity improves the short-distance behavior of gravitational fields, with unchanged asymptotics.
- There are unexpected consequences of non-locality, too, for example in mini-black hole formation.

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Open questions:

- Are there exact, singularity-free solutions?
- Perturbative techniques in non-local QFT?
- Non-locality in cosmological scenarios?

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Thank you for your attention :)

Abstract

Ultrarelativistic objects in non-local infinite-derivative gravity

Einstein's General Theory of Relativity (GR) has proven a remarkably accurate description of gravitation at the very large scales. At small scales and high energy densities, however, it is plagued by singularities: these are regions of space and time where the spacetime curvature diverges, hence depriving GR of its predictive power. It is believed that a suitable UV completion of classical gravity will solve that problem, a task that has proven difficult if treated at the full quantum level.

I will present a classical, Lorentz-invariant, but non-local modification of General Relativity that becomes important at a small length scale L. Treated at the linear level the presence of non-locality indeed resolves the singularity problem. However, one may ask whether it is justified to take these linear results seriously. In this talk I will explain one argument in favor:

It has long been shown that the linearized gravitational field of a particle of mass m, when boosted to the speed of light in a suitable limit, describes an exact solution of Einstein's NON-linear field equations. In this talk I will derive the gravitational field of an ultrarelativistic object for non-local gravity in a similar fashion, with the conjecture that it may also solve the non-linear equations (but that still needs to be shown). If time permits I will comment on interesting consequences for mini-black hole formation.

Based on: Jens Boos, Jose Pinedo Soto, and Valeri P. Frolov, “Ultrarelativistic spinning objects (gyratons) in non-local ghost-free gravity,” arXiv:2004.07420 [gr-qc], to appear in Phys. Rev. D.