Linearized short-distance modifications of Einstein's General Relativity

Jens Boos (supervisor: Prof. Valeri P. Frolov)
boos@ualberta.ca
University of Alberta

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Einstein's General Relativity is a complicated non-linear theory.

The Einstein field equations describe how spacetime and matter are intertwined:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^3} T_{\mu\nu}$$

They are ten non-linear partial differential equations for the ten functions $g_{\mu\nu}(x)$, if one provides the energy-momentum (energy density, pressure, shear,...) $T_{\mu\nu}(x)$.

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There exist many exact solutions for the Einstein equations:

- flat spacetime: Minkowski space (particle physics)
- black hole spacetimes (rotating, charged, accelerating, with cosmological constant)
- the entire Universe (cosmology)

In this talk: let us understand the gravitational field of a **point particle**.

Problem: Tidal forces diverge close to point particles.

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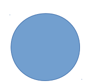
point particle
of mass **m**



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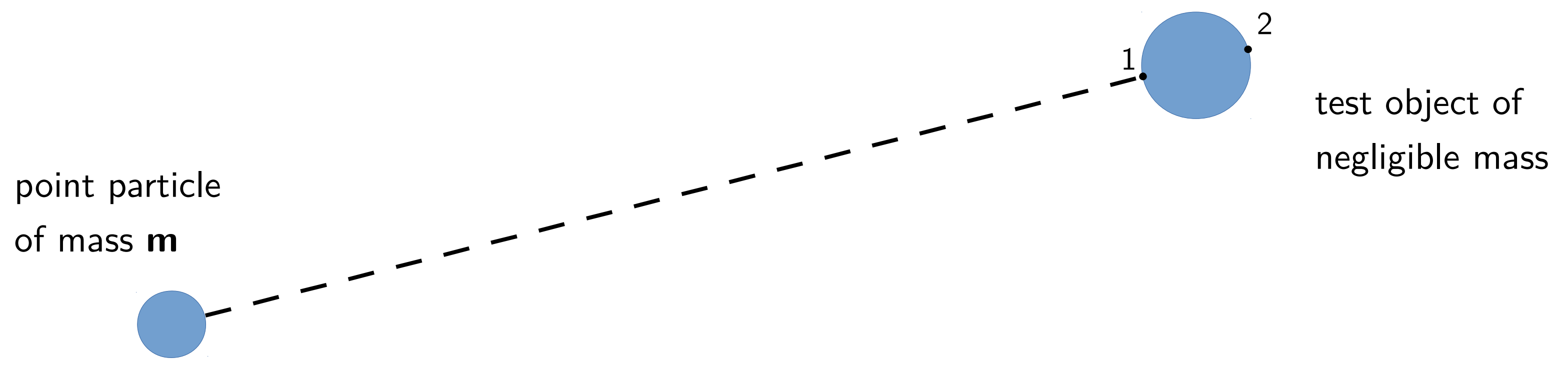

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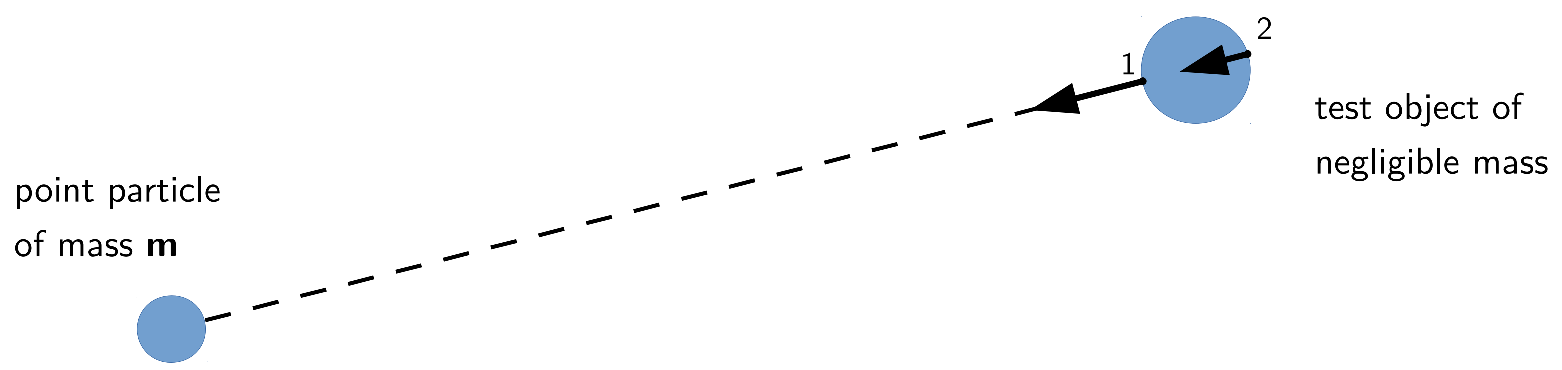

Forces at points 1 and 2: $F_1 = -\frac{Gm}{r^2},\ F_2 = -\frac{Gm}{(r+\Delta r)^2},\ F_2 - F_1 \approx \frac{2Gm}{r^3}\Delta r + \mathcal{O}(\Delta r^2)\,.$

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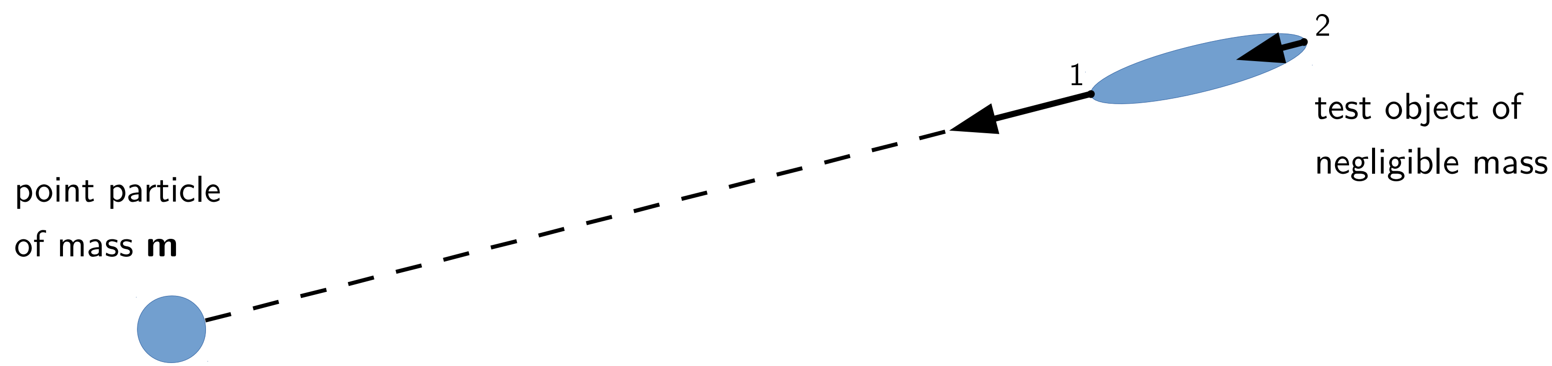

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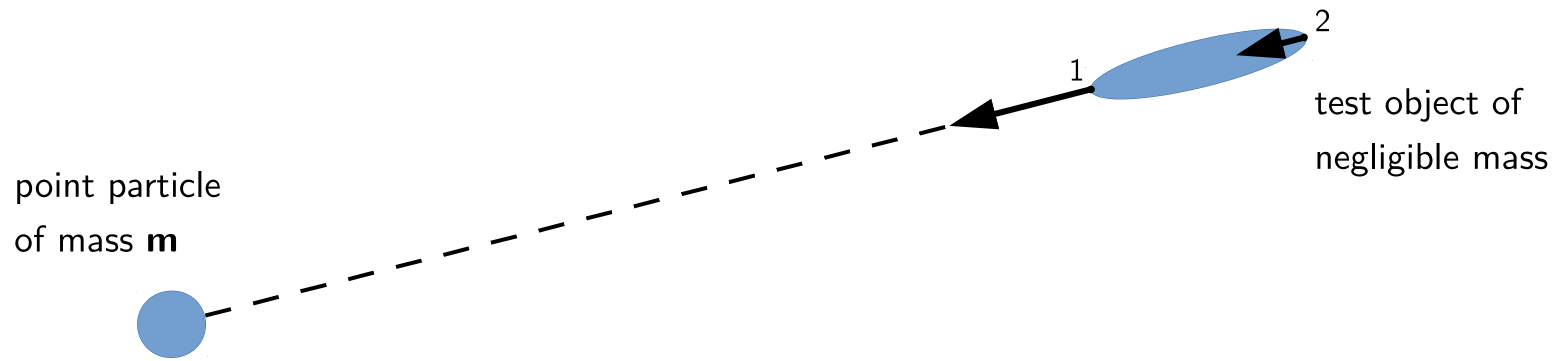
Spacetime curvature can be thought of as the tidal force (tensor) on an object:



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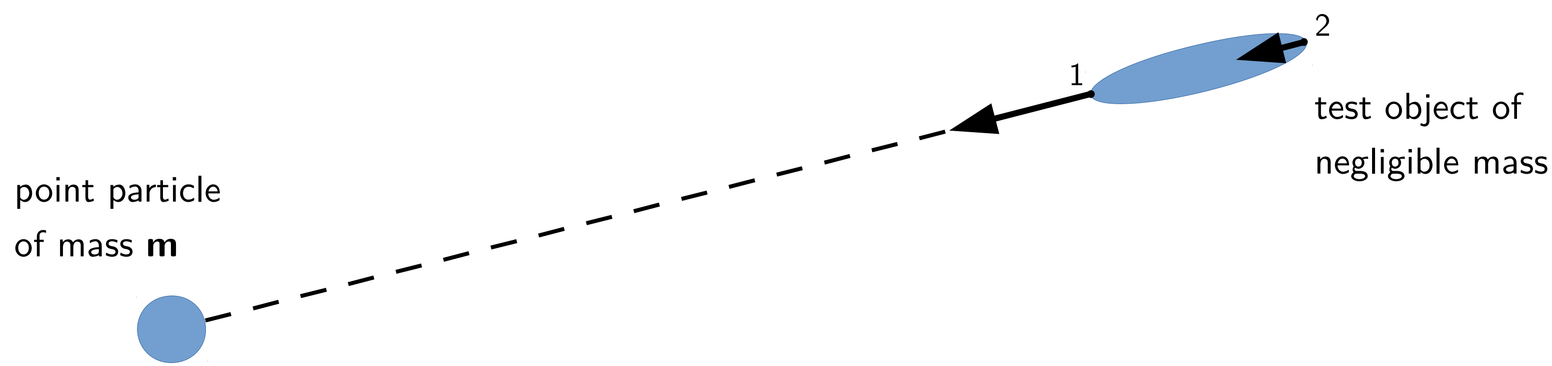

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Important observation: the tidal force **diverges** for $r \to 0$! This is a simple example of the presence of unphysical “singularities” in General Relativity.

Root of all evil? The gravitational potential of a point particle is singular, $\phi = -\frac{Gm}{r}$.

Solution? $\to$ Need to modify General Relativity at **small scales** only.

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For ghost-free gravity (one possible modification), we take the Einstein equations

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and insert the following **non-local operators** (at the linear level):

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Here, $\Box = -\frac{\partial^2}{c^2 \partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the d'Alembert operator, and $f(\Box) = e^{-\Box/\mu^2}$.

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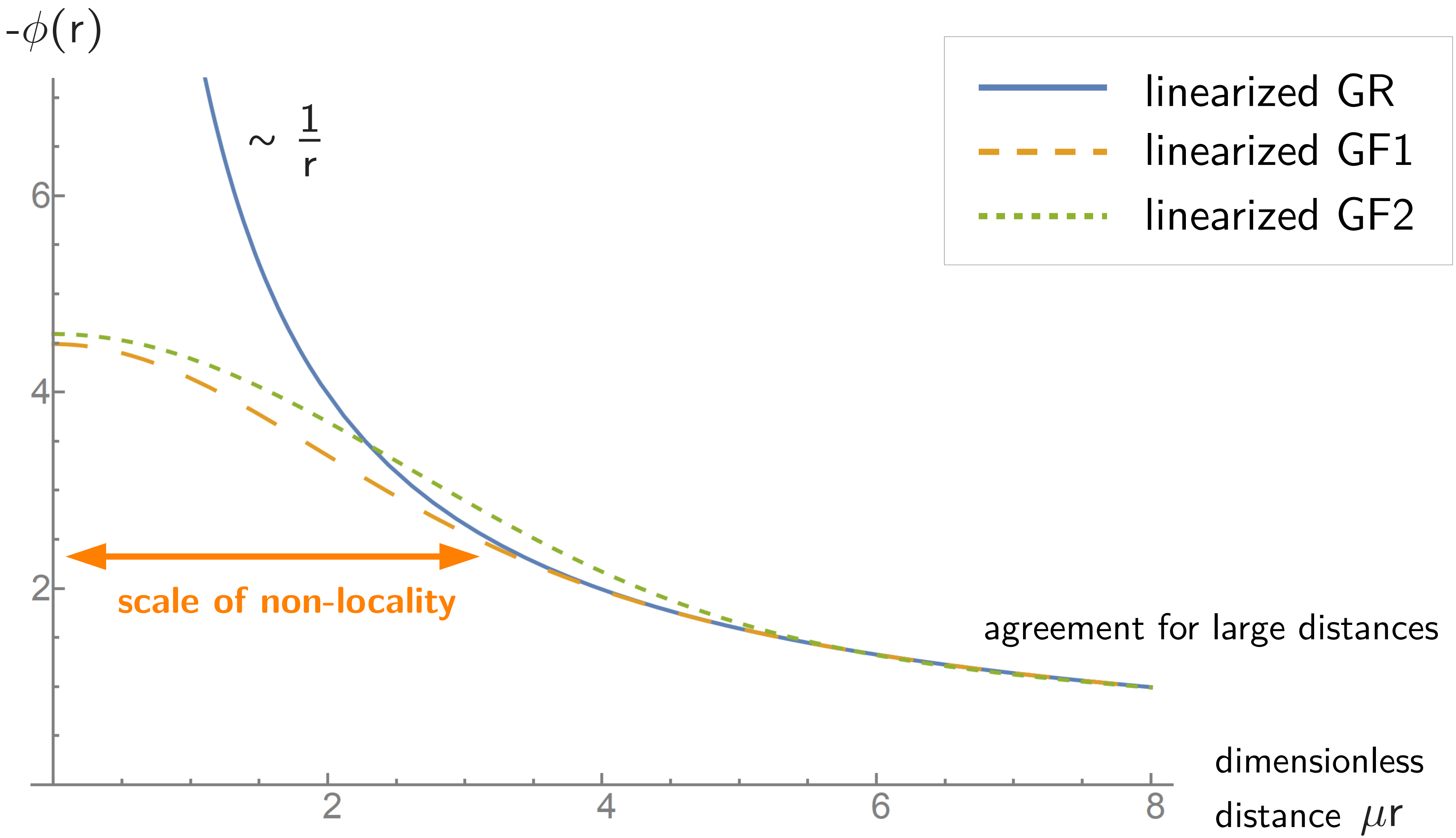
This theory happens to be **non-local**: $e^{a\partial_x} f(x) \equiv \left(\sum_{n=0}^{\infty} \frac{a^n}{n!} \partial_x^n \right) f(x) = f(x+a)$

Point particles in lin. ghost-free gravity have no singularities.



Based on J.B., Valeri P. Frolov, and Andrei Zelnikov, arXiv:1802.09573 [gr-qc].

Thank you for your attention.