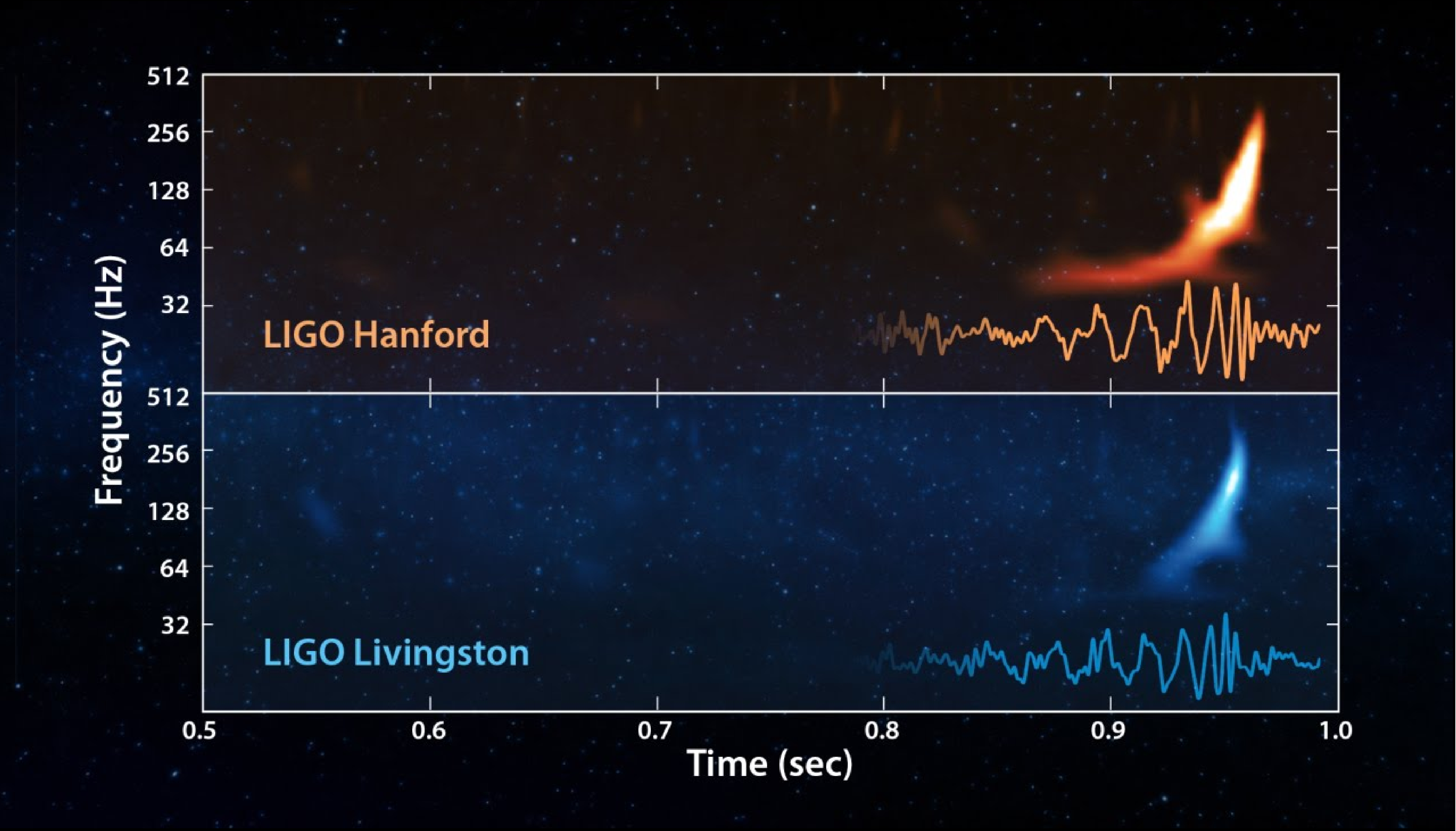
Quasi-normal modes: what can ringing black holes tell us about quantum gravity?

Jens Boos (supervisor: Prof. Valeri P. Frolov)
boos@ualberta.ca
University of Alberta

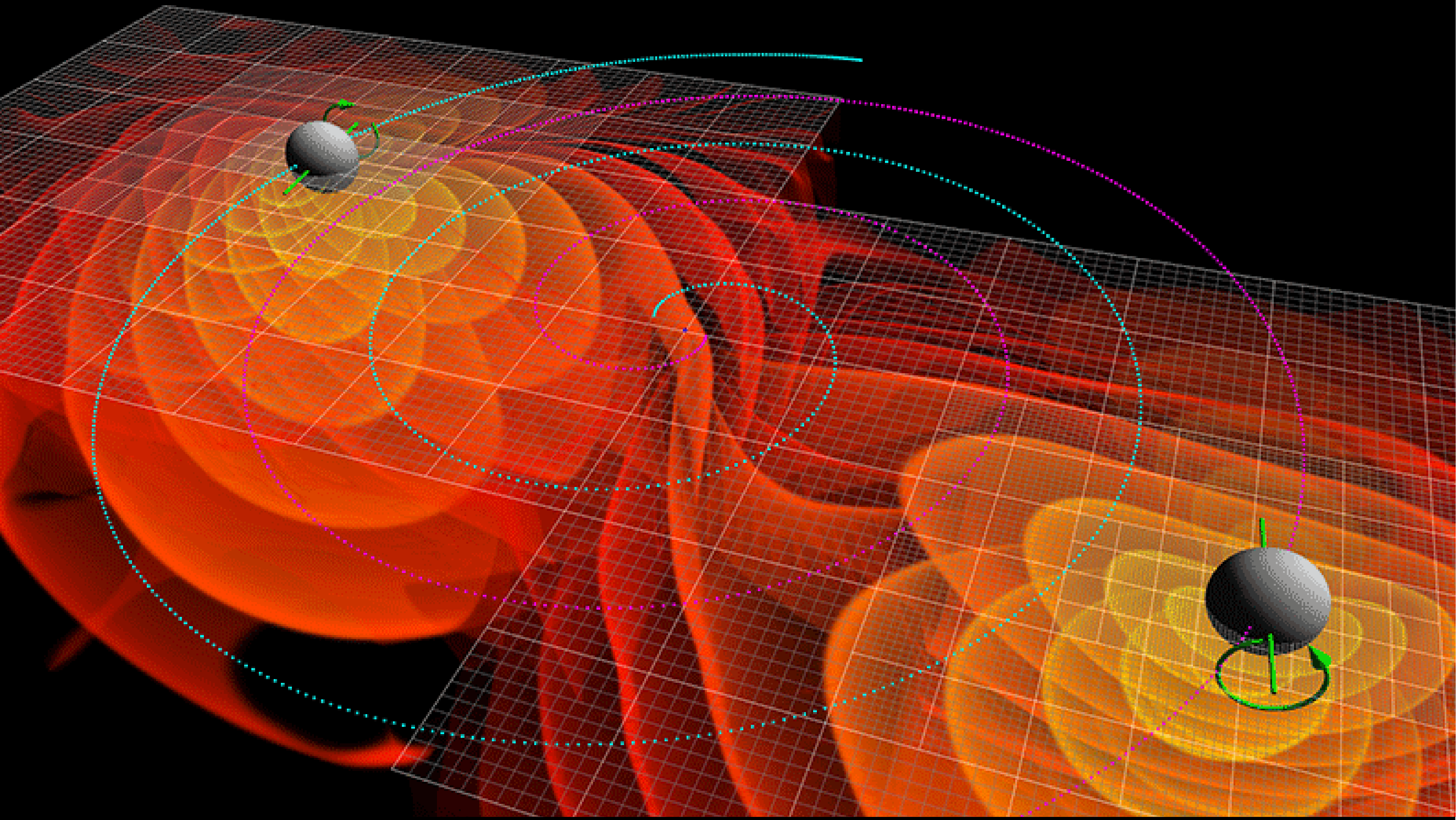
Friday, Sept 23, 2016, 13:00

The Seventh Annual Symposium for Graduate Physics Research, University of Alberta

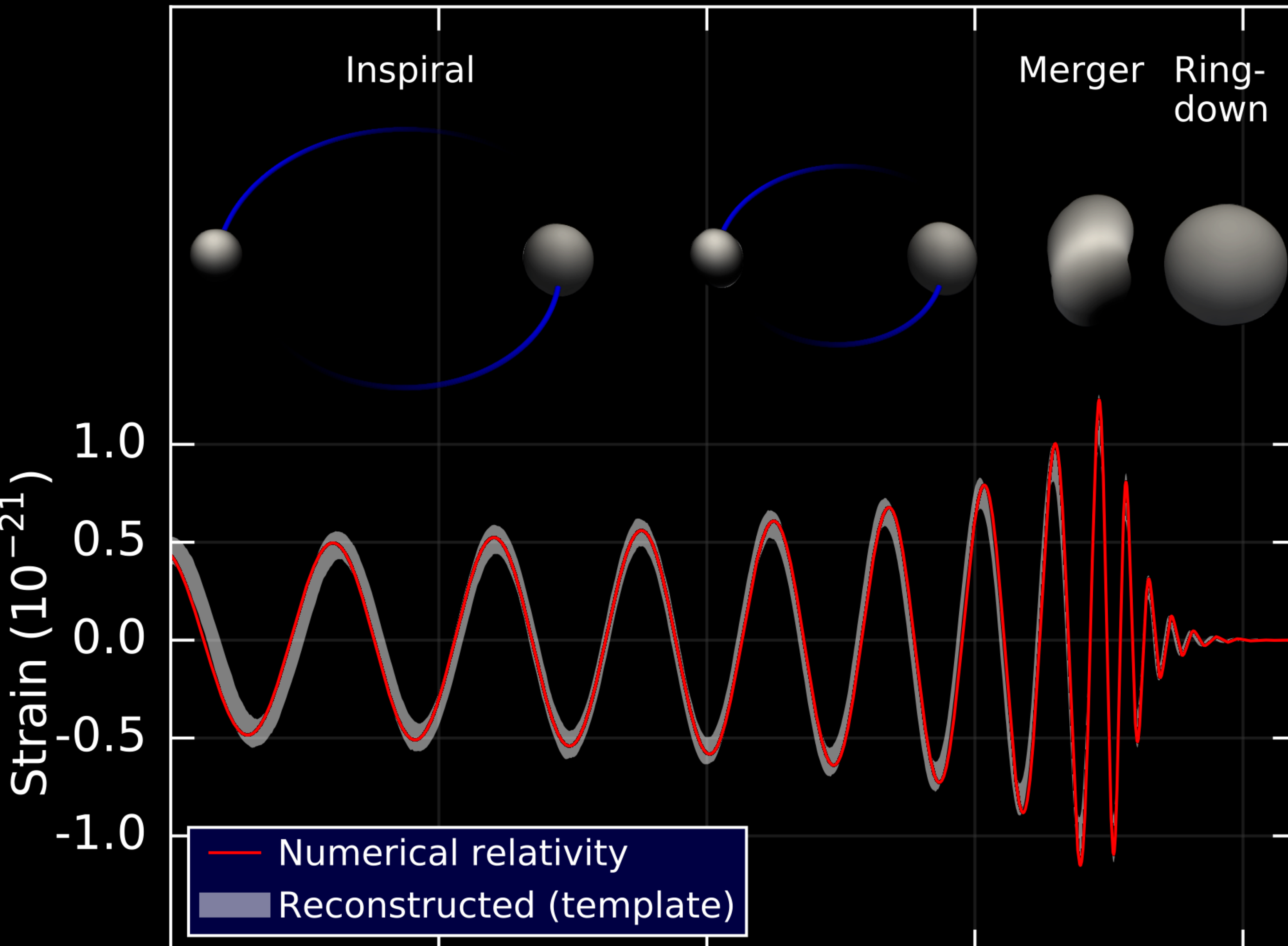
Introducing: gravitational wave GW150914



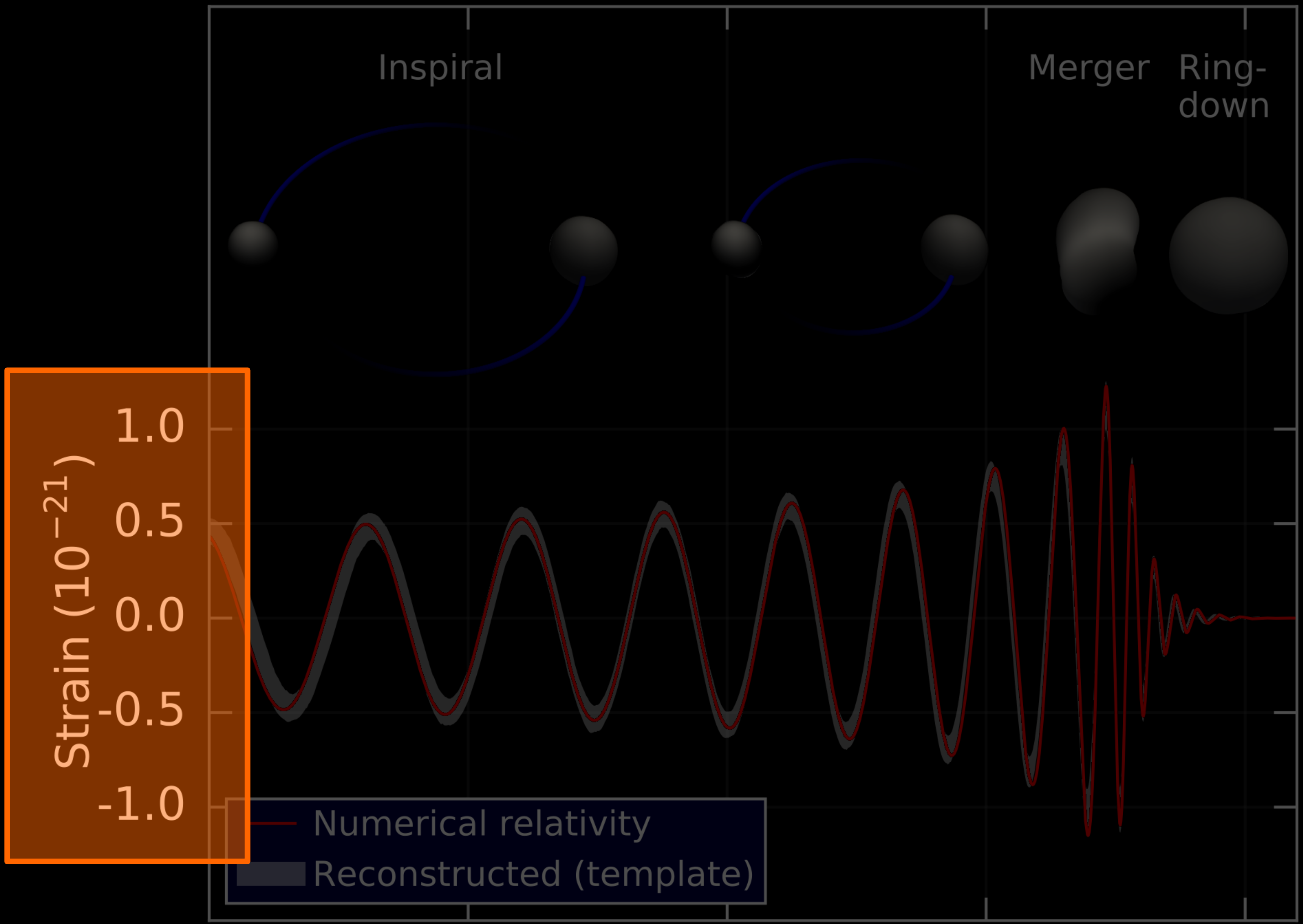
GW150914 = colliding black holes!



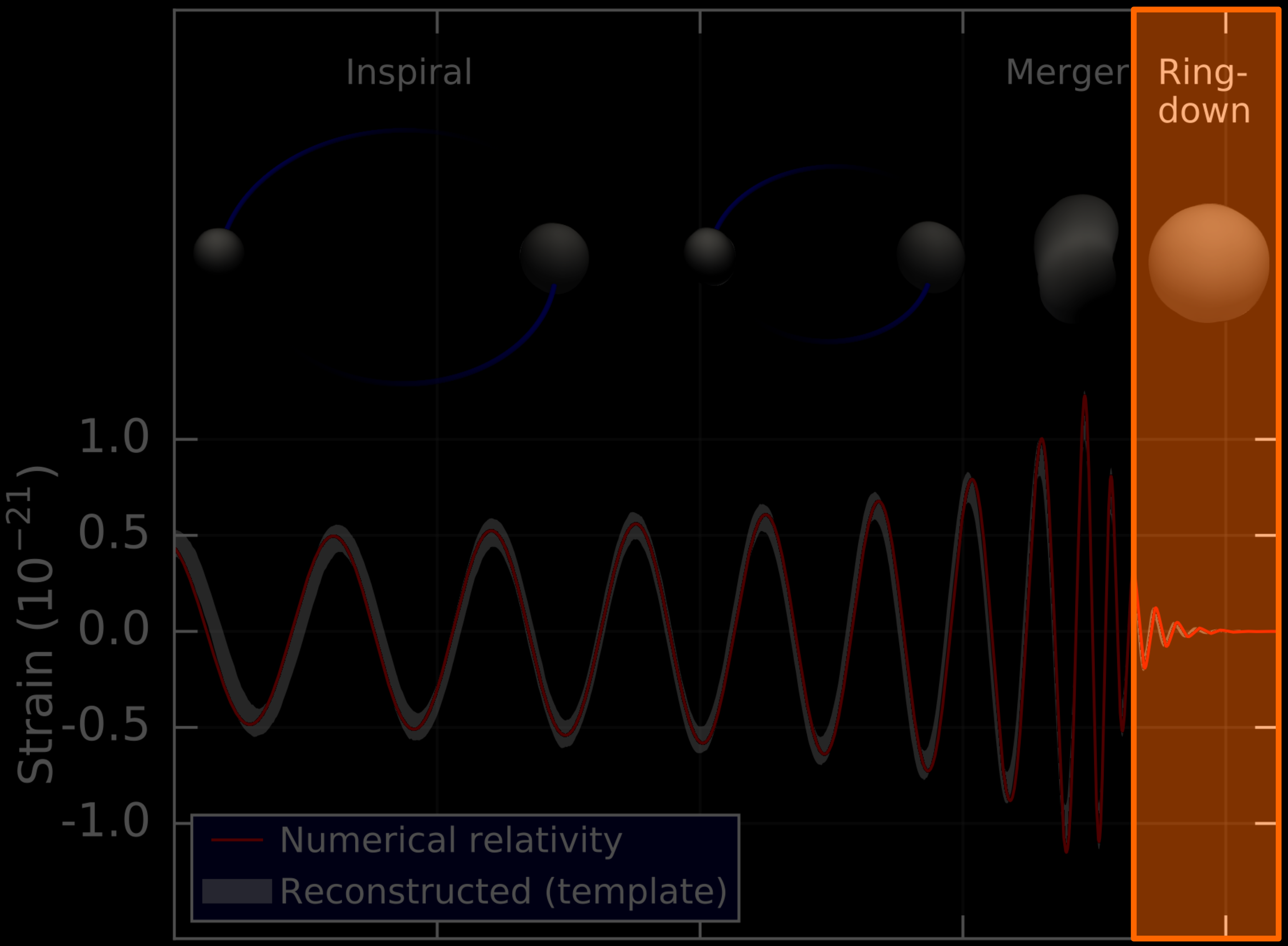
How is that related to quasi-normal modes?



How is that related to quasi-normal modes?



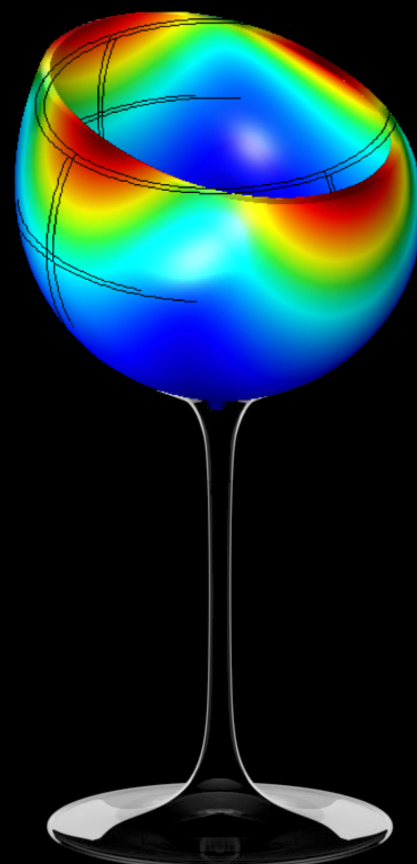
How is that related to quasi-normal modes?



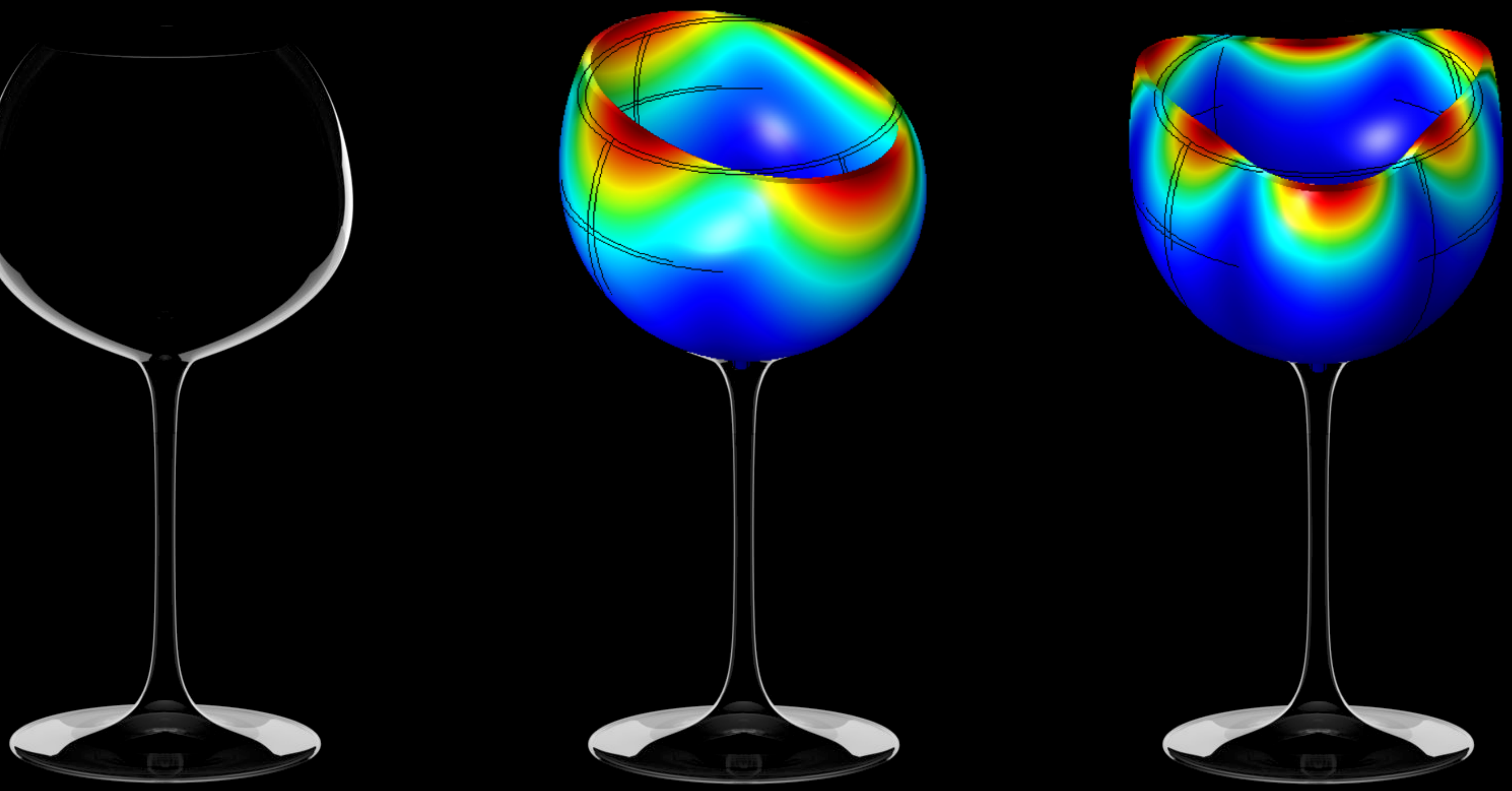
What are quasi-normal modes? (1/2)

What are quasi-normal modes? (1/2)

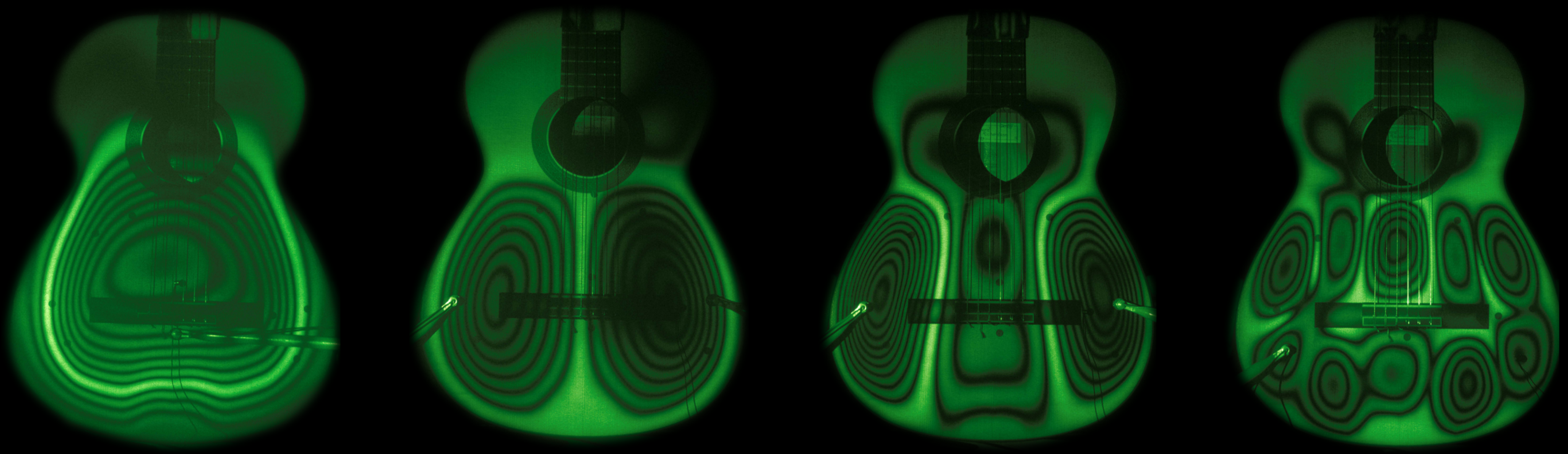
What are quasi-normal modes? (1/2)



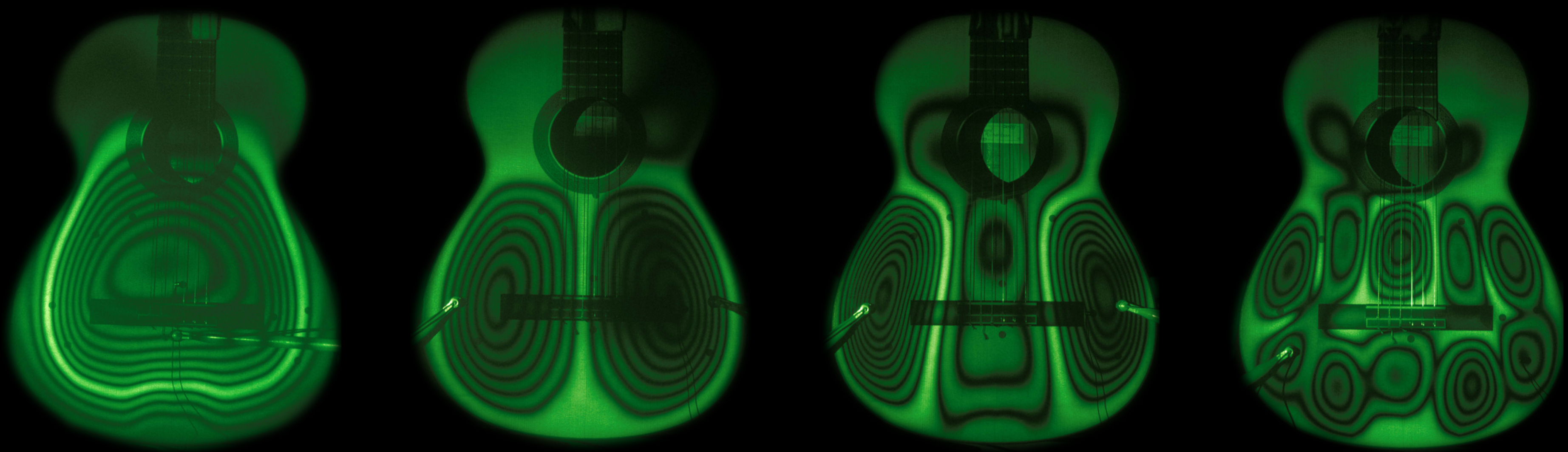
What are quasi-normal modes? (1/2)



What are quasi-normal modes? (2/2)

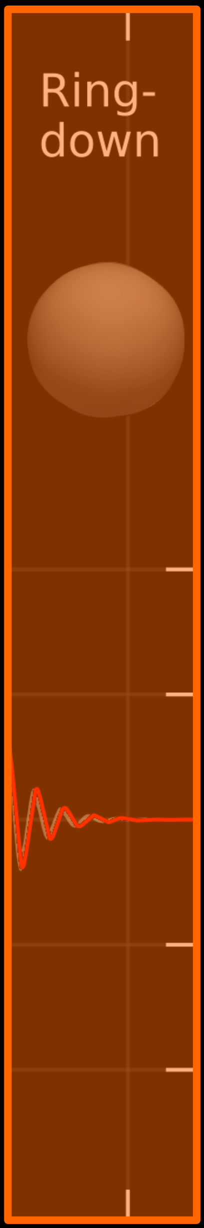


What are quasi-normal modes? (2/2)



Quasi-normal modes = standing waves + dissipation.

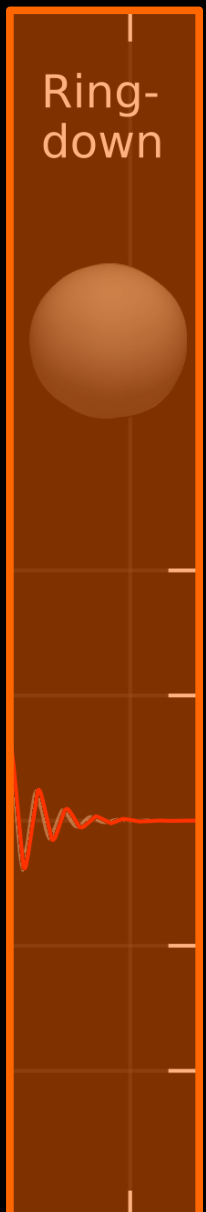
How do we describe quasi-normal modes mathematically? (1/3)



Approximate amplitude as $A \sim \sin(\omega t) \times \exp(-\lambda t)$.

$\rightarrow$ corresponds to $\omega \in \mathbb{C}$ with $A \sim \exp(-i\omega t)$.

How do we describe quasi-normal modes mathematically? (1/3)



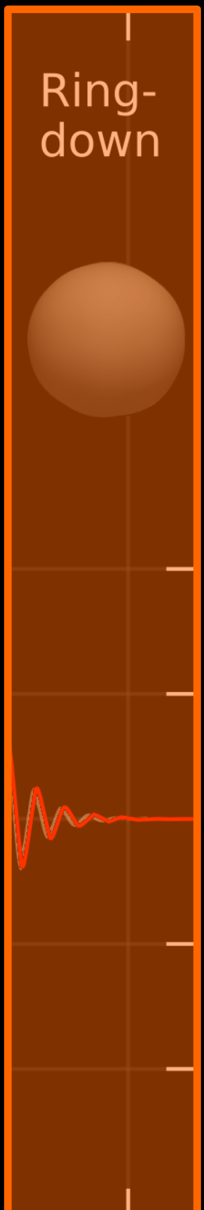
Approximate amplitude as $\mathsf{A} \sim \sin(\omega t) \times \exp(-\lambda t)$.

$\rightarrow$ corresponds to $\omega \in \mathbb{C}$ with $\mathsf{A} \sim \exp(-i\omega t)$.

Where does the amplitude come from?

$\rightarrow$ solution of wave equation

How do we describe quasi-normal modes mathematically? (1/3)



Approximate amplitude as $A \sim \sin(\omega t) \times \exp(-\lambda t)$.

$\rightarrow$ corresponds to $\omega \in \mathbb{C}$ with $A \sim \exp(-i\omega t)$.

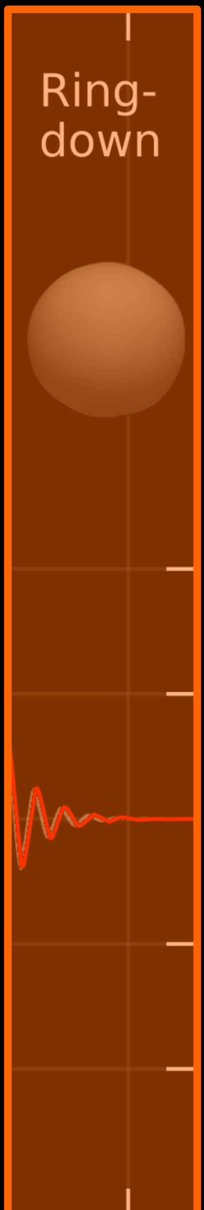
Where does the amplitude come from?

$\rightarrow$ solution of wave equation

Application to black holes?

$\rightarrow$ solve wave equation $c^2 \nabla^2 \Phi = \partial_t^2 \Phi$ “in black hole background”

How do we describe quasi-normal modes mathematically? (1/3)



Approximate amplitude as $A \sim \sin(\omega t) \times \exp(-\lambda t)$.

$\rightarrow$ corresponds to $\omega \in \mathbb{C}$ with $A \sim \exp(-i\omega t)$.

Where does the amplitude come from?

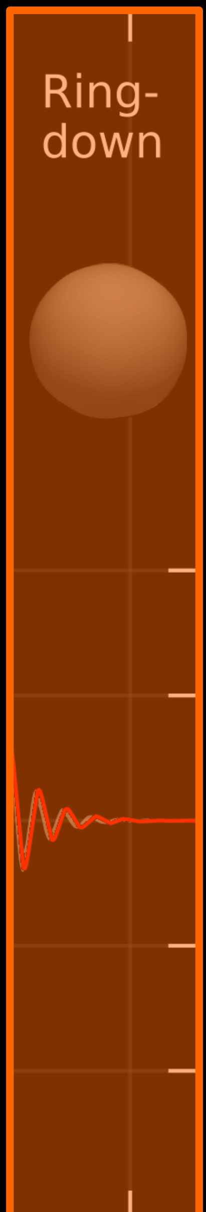
$\rightarrow$ solution of wave equation

Application to black holes?

$\rightarrow$ solve wave equation $c^2 \nabla^2 \Phi = \partial_t^2 \Phi$ "in black hole background"

Question (used to be): Is the black hole stable?

How do we describe quasi-normal modes mathematically? (1/3)



Approximate amplitude as $A \sim \sin(\omega t) \times \exp(-\lambda t)$.

$\rightarrow$ corresponds to $\omega \in \mathbb{C}$ with $A \sim \exp(-i\omega t)$.

Where does the amplitude come from?

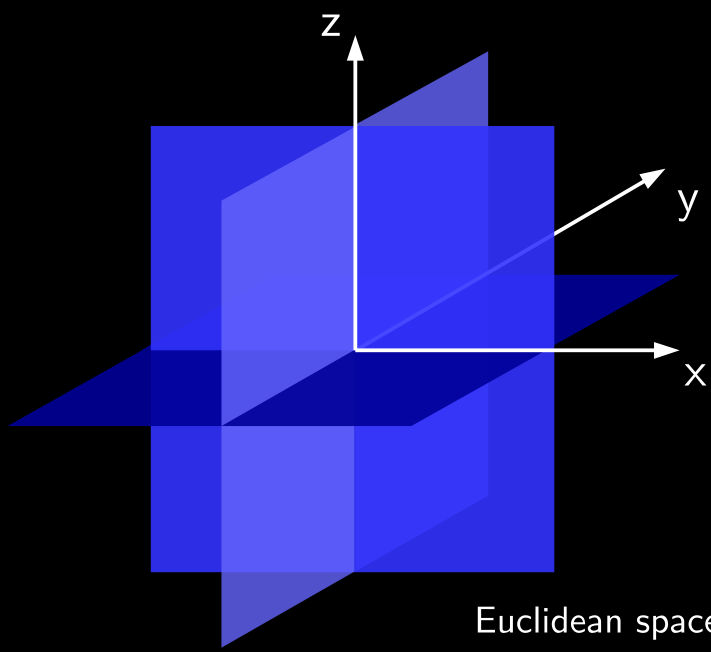
$\rightarrow$ solution of wave equation

Application to black holes?

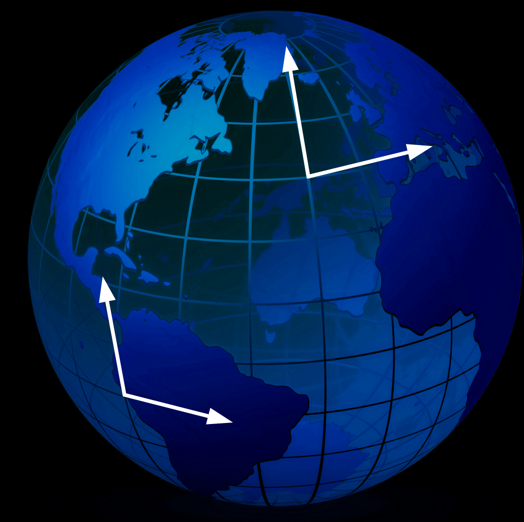
$\rightarrow$ solve wave equation $c^2 \nabla^2 \Phi = \partial_t^2 \Phi$ "in black hole background"

Question (now): What can we learn about quantum gravity?

How do we describe quasi-normal modes mathematically? (2/3)

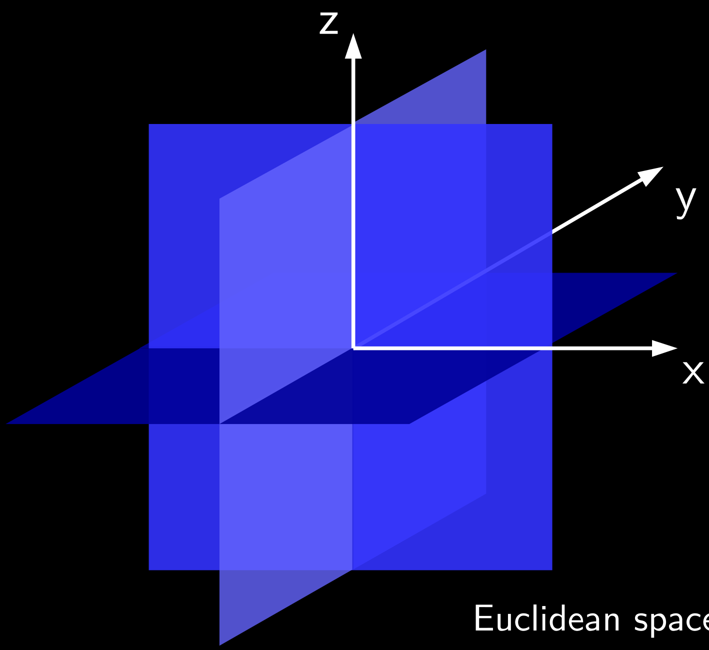


Euclidean space



the 2-sphere

How do we describe quasi-normal modes mathematically? (2/3)



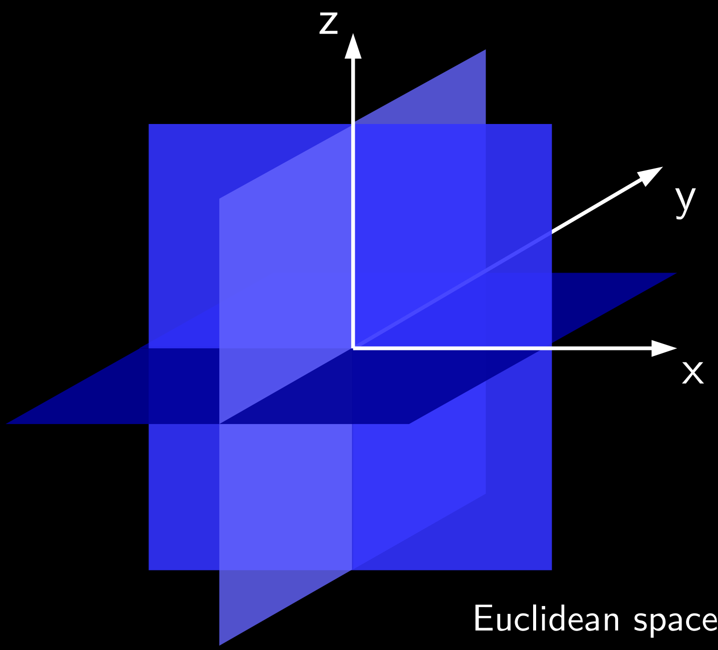
Euclidean space

the 2-sphere

$$\nabla^2\Phi = \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\Phi$$

$$\nabla^2\Phi = \frac{1}{\sin\varphi}\partial_\varphi\left(\sin\varphi\,\partial_\varphi\Phi\right) + \frac{1}{\sin^2\varphi}\partial_\theta^2\Phi$$

How do we describe quasi-normal modes mathematically? (2/3)



Euclidean space

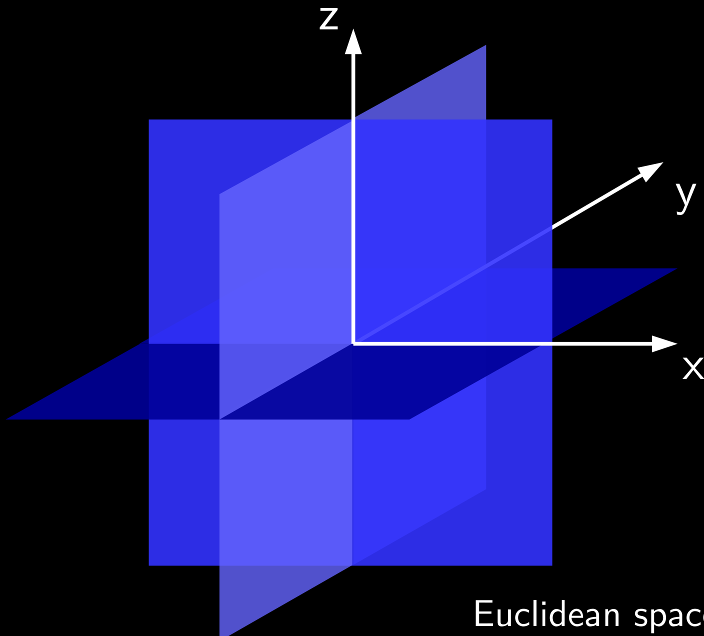
the 2-sphere

$$\nabla^2\Phi = \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\Phi$$

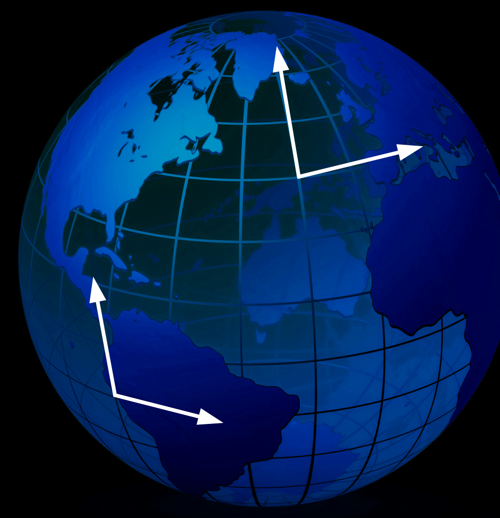
$$\nabla^2\Phi = \frac{1}{\sin\varphi}\partial_\varphi\left(\sin\varphi\,\partial_\varphi\Phi\right) + \frac{1}{\sin^2\varphi}\partial_\theta^2\Phi$$

$\rightarrow$ the wave operators know about the geometry of spacetime

How do we describe quasi-normal modes mathematically? (2/3)



Euclidean space



the 2-sphere

$$\nabla^2\Phi = \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\Phi$$

$$\nabla^2\Phi = \frac{1}{\sin\varphi}\partial_\varphi\left(\sin\varphi\,\partial_\varphi\Phi\right) + \frac{1}{\sin^2\varphi}\partial_\theta^2\Phi$$

$\rightarrow$ the wave operators know about the geometry of spacetime

$\rightarrow$ quasi-normal modes encode geometric properties of spacetime

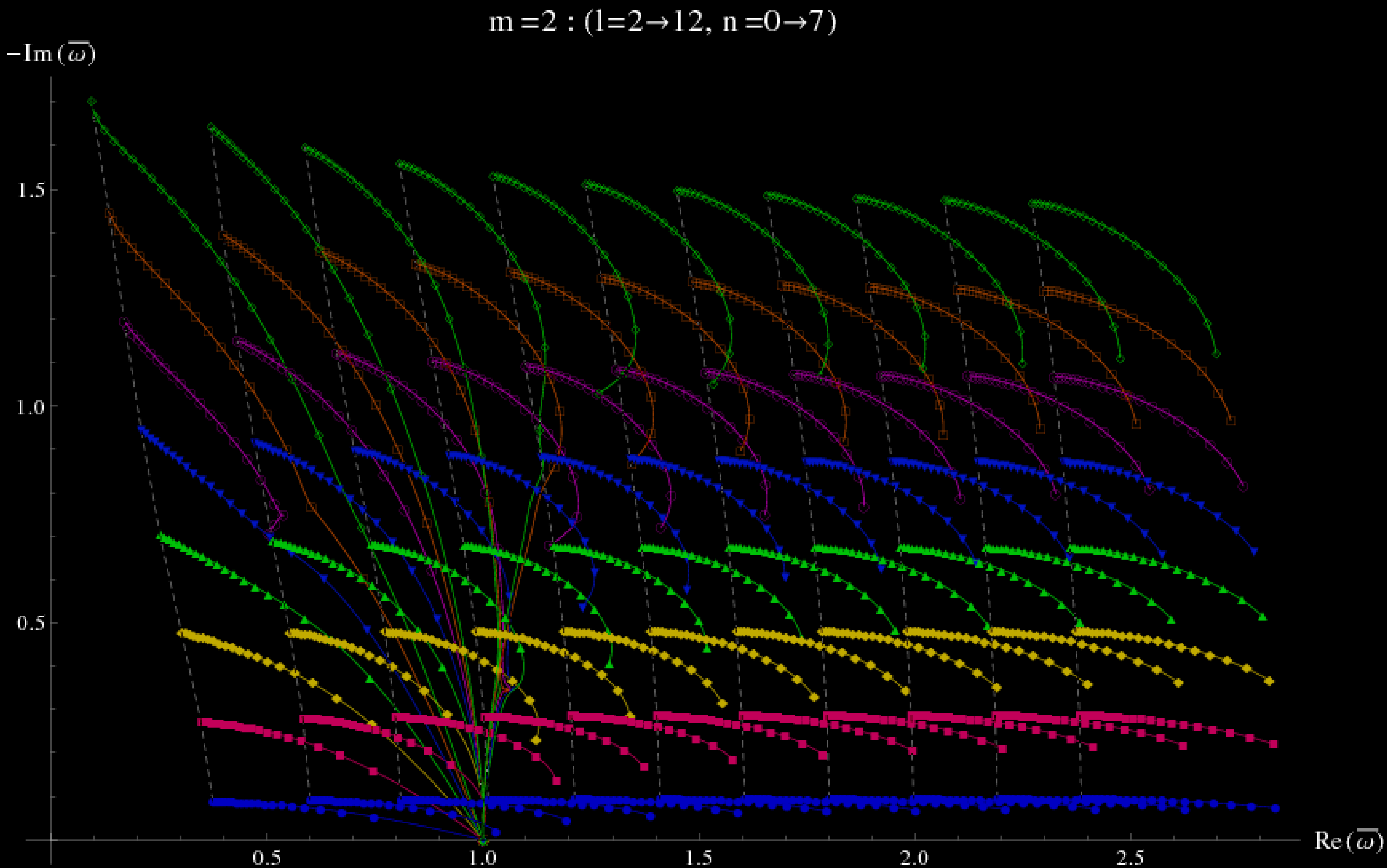
How do we describe quasi-normal modes mathematically? (3/3)

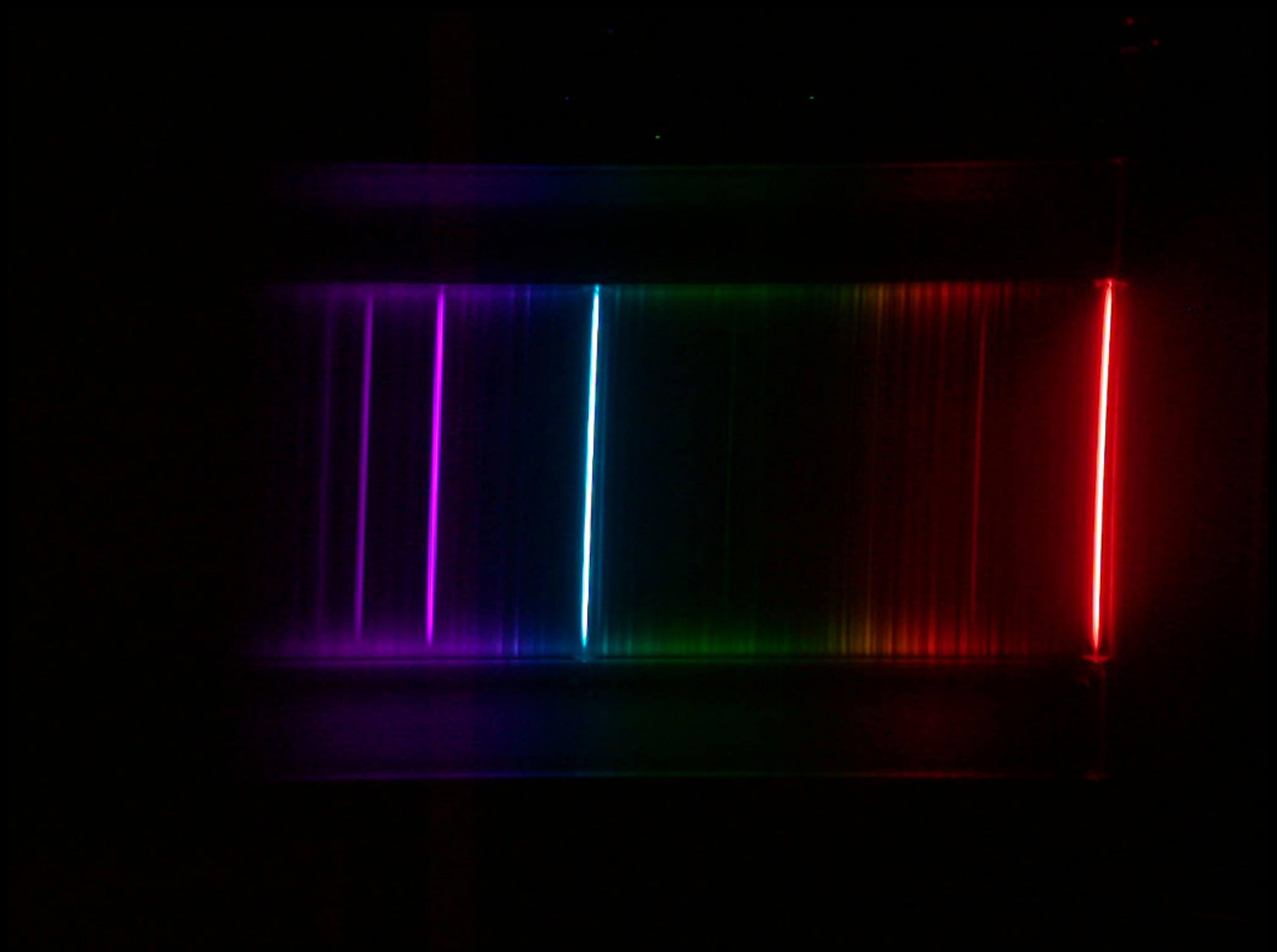
rotating black hole (mass M, angular momentum Ma)

$$\nabla^2\Phi = \Big\{ \frac{1}{\Delta}\left[(r^2+a^2)^2 - \Delta a^2 \sin^2\theta\right]\partial_t^2 - \partial_r\left(\Delta\partial_r\right) - \frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\,\partial_\theta\right)$$
$$- \frac{1}{\Delta\sin^2\theta}\left(\Delta - a^2\sin^2\theta\right)\partial_\varphi^2 + \frac{2a}{\Delta}\left[(r^2+a^2)-\Delta\right]\partial_t\partial_\varphi \Big\}\Phi,$$

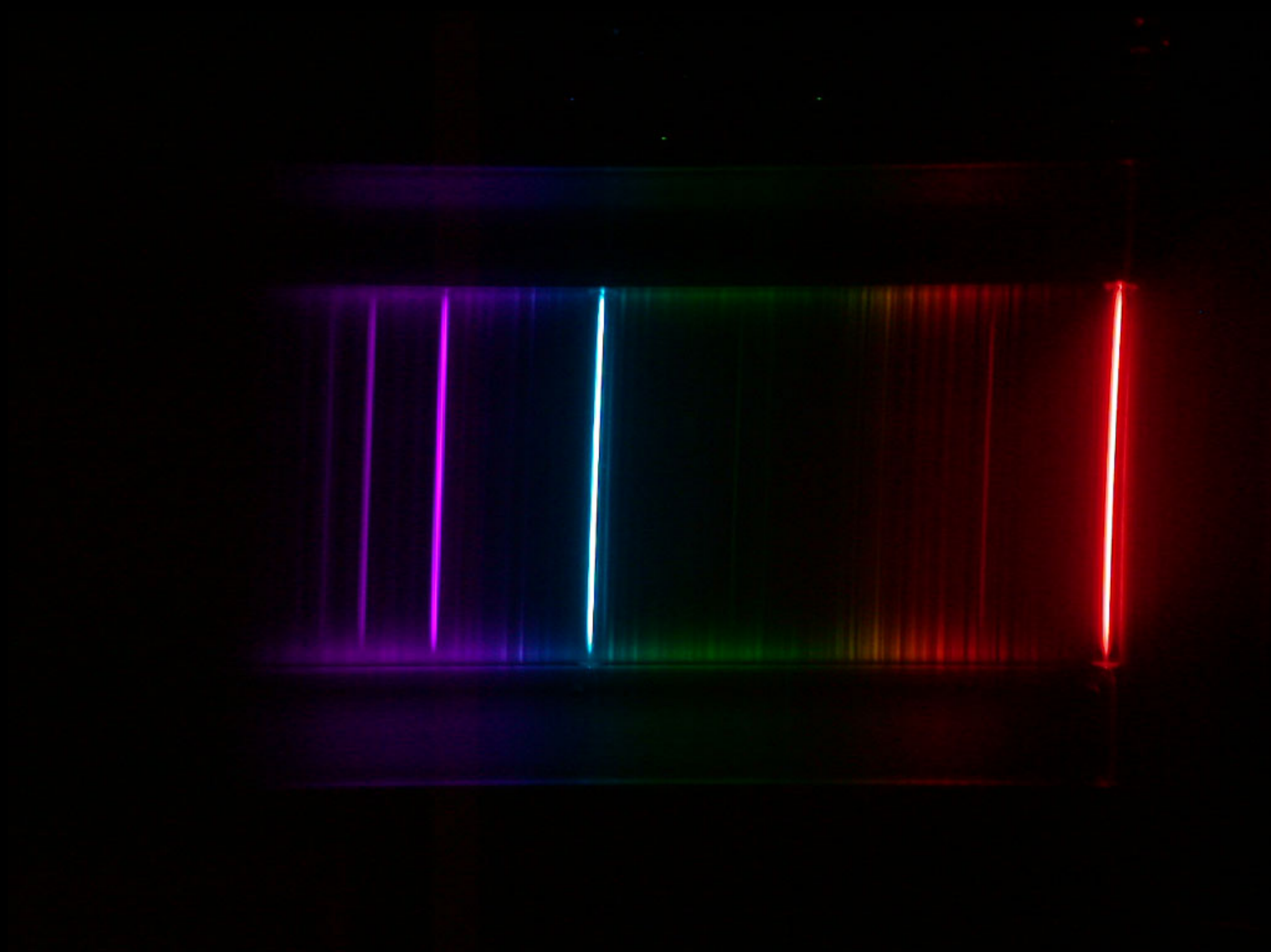
where the black hole parameters M and a hide in $\Delta := r^2 - 2Mr - a^2$.

The result: black hole quasi-normal modes are discrete!

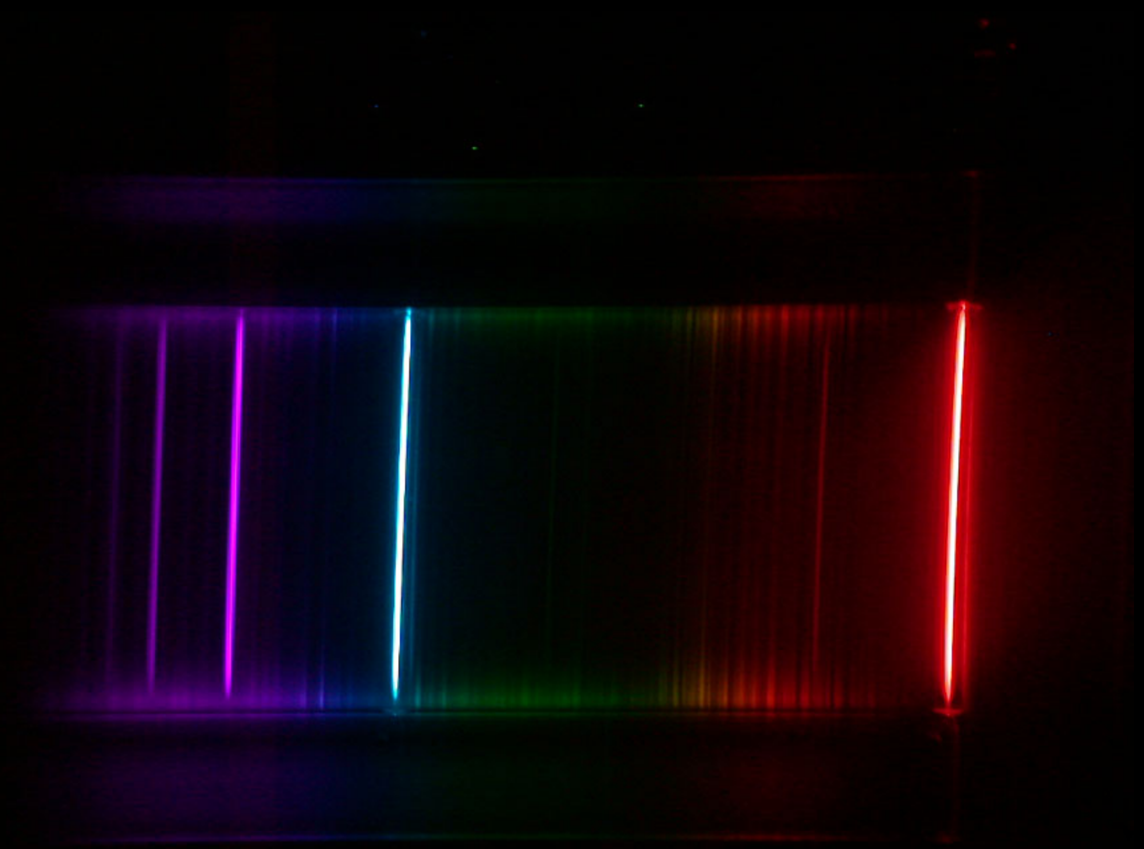




Useful analogy: quantum behavior of hydrogen

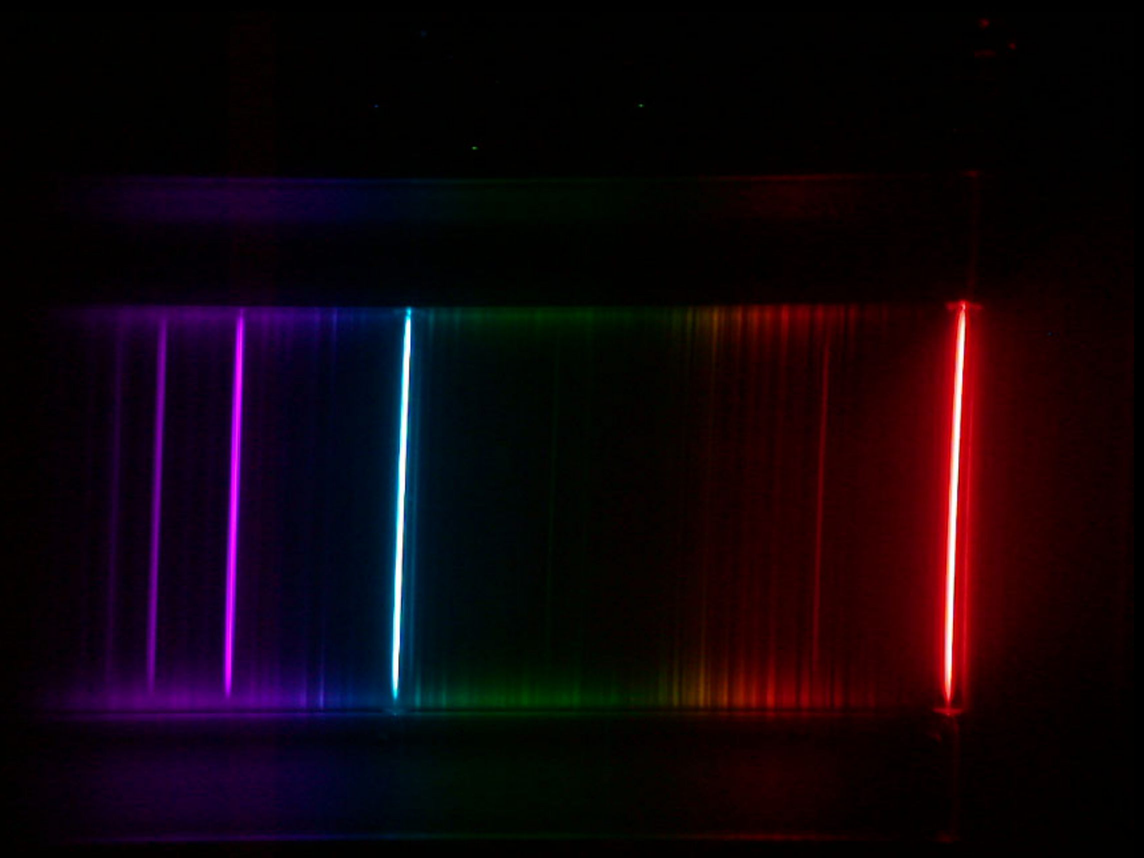


Useful analogy: quantum behavior of hydrogen



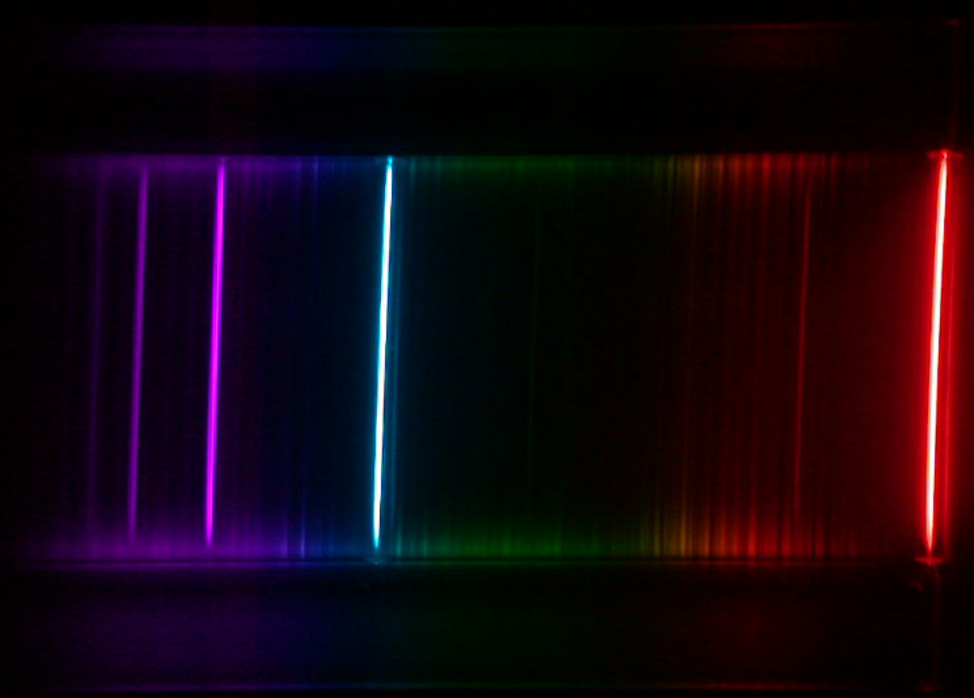
$$E_{n,j} \approx -\frac{m_e c^2 \alpha^2}{2n^2}\left[1 + \frac{\alpha^2}{n^2}\left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4}\right)\right] \approx -\frac{m_e c^2 \alpha^2}{2n^2}$$

Useful analogy: quantum behavior of hydrogen



→ Can we think of black holes as the “atoms of quantum gravity”?

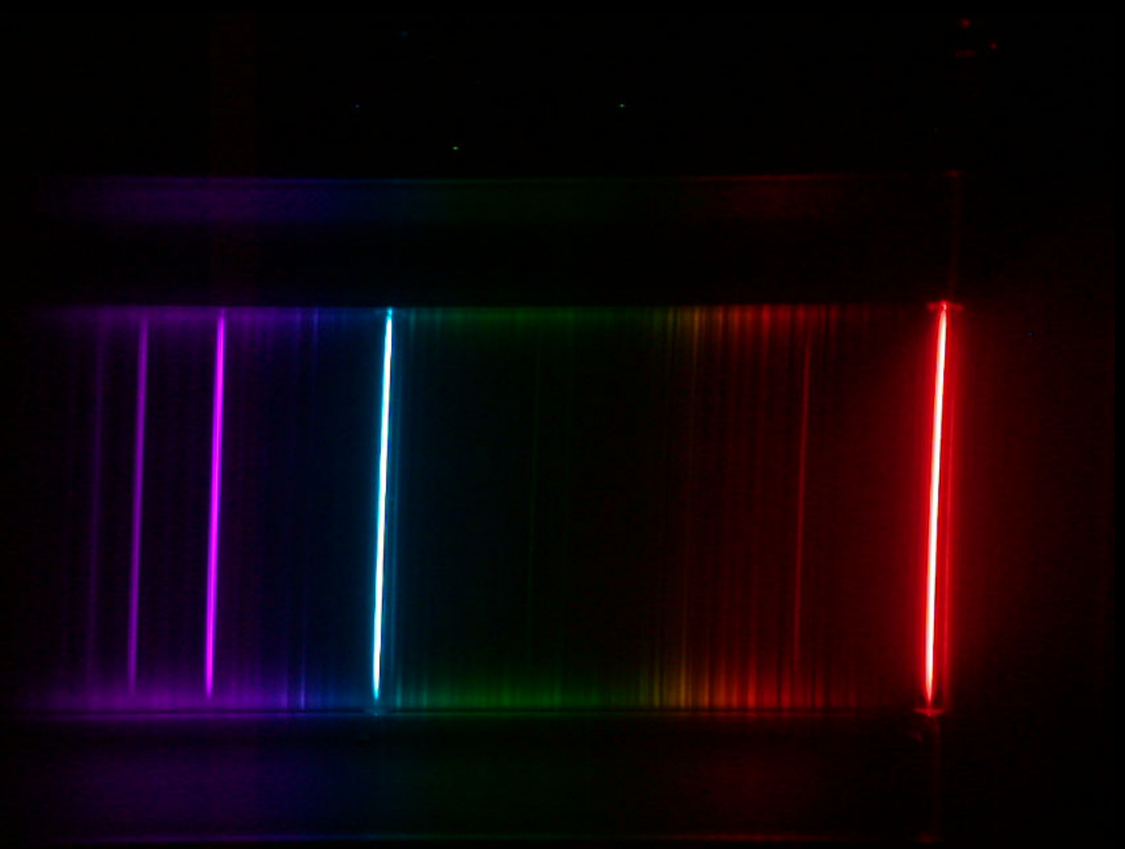
Useful analogy: quantum behavior of hydrogen



→ Can we think of black holes as the "atoms of quantum gravity"?

→ Will their discrete "spectrum" help us to find clues about quantum gravity?

Useful analogy: quantum behavior of hydrogen



→ Can we think of black holes as the "atoms of quantum gravity"?

→ Will their discrete "spectrum" help us to find clues about quantum gravity?

Thank you for your attention.

List of Figures

- http://interstellarfilm.wikia.com/wiki/Gargantua
- Georgia Tech, http://www.youtube.com/watch?v=TWqhUANNFXw
- C. Henze/NASA Ames Research Center via http://physics.aps.org/articles/v9/17
- B.P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration)
- Teruyuki Kozuka via http://www.comsol.com/paper/vibration-analysis-of-the-Wineglass-for-glass-harp-using-comsol-multiphysics-17261
- http://www.bbc.co.uk/news/science-environment-13573631
- http://en.wikipedia.org/wiki/Black_hole_information_paradox
- G. B. Cook and M. Zalutskiy, “Gravitational perturbations of the Kerr geometry: High-accuracy study,” Phys. Rev. D **90** (2014) no.12, 124021, arXiv:1410.7698.
- Brian K. Niece, "Simultaneous Display of Spectral Images and Graphs Using a Web Camera and Fiber-Optic Spectrophotometer,” J. Chem. Educ. **83** (2006) 761 via http://astro.psu.edu/public-outreach/fireworks-masks-1/sample-spectra-of-gases

Abstract

The recent detection of gravitational waves [1] is truly mind-boggling: ripples in spacetime itself were directly detected for the first time, with an instrument sensitive enough to measure dislocations the size of an atomic nucleus over a distance of a few kilometers. This discovery can also be considered the first direct detection of black holes.

In this talk, we will discuss one aspect of black hole physics: so-called quasi-normal modes [2]. These are characteristic frequencies emitted by black holes when they are subject to perturbations: much like the ringing of a wine glass, when struck by a solid object.

We will describe how to calculate quasi-normal modes, and in a second step elucidate as to what information these frequencies may contain. As it turns out, if measured precisely enough, they might be able to give us crucial insight into the still elusive quantum theory of gravity [3].

References

[1] B. P. Abbott et al. [LIGO Scientific and Virgo Collaborations], "Observation of Gravitational Waves from a Binary Black Hole Merger," Phys. Rev. Lett. **116** 061102 (2016) no. 6, arXiv: 1602.03837 [gr-qc].

[2] K. D. Kokkotas and B. G. Schmidt, "Quasi-Normal Modes of Stars and Black Holes", Living Rev. Relativity **2** 2 (1999).

[3] C. Corda, "Quasi-Normal Modes: The 'Electrons' of Black Holes as 'Gravitational Atoms'? Implications for the Black Hole Information Puzzle," Adv. High Energy Phys. **2015** 867601 (2015), arXiv:1503.00565 [gr-qc].