Non-local “ghost-free” gravity

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Based on...

5. JB, V. P. Frolov and A. Zelnikov, “On thermal field fluctuations in ghost-free theories,” Phys. Lett. B **793** (2018) 290 ; 1904.07917 [hep-th].
4. JB, V. P. Frolov and A. Zelnikov, “Probing the vacuum fluctuations in scalar ghost-free theories,” Phys. Rev. D **99**, no. 7 (2019) 076014; 1901.07096 [hep-th].
3. JB, V. P. Frolov and A. Zelnikov, “Quantum scattering on a delta potential in ghost-free theory,” Phys. Lett. B **782** (2018) 688; 1805.01875 [hep-th].
2. JB, “Gravitational Friedel oscillations in higher-derivative and infinite-derivative gravity?,” Int. J. Mod. Phys. D **27** (2018) 1847022; 1804.00225 [gr-qc]
1. JB, V. P. Frolov and A. Zelnikov, “The gravitational field of static p-branes in linearized ghost-free gravity,” Phys. Rev. D **97**, no. 8 (2018) 084021; 1802.09573 [gr-qc].

Motivation and overview

It takes an **infinite** amount of work to move a point particle on to another mass.

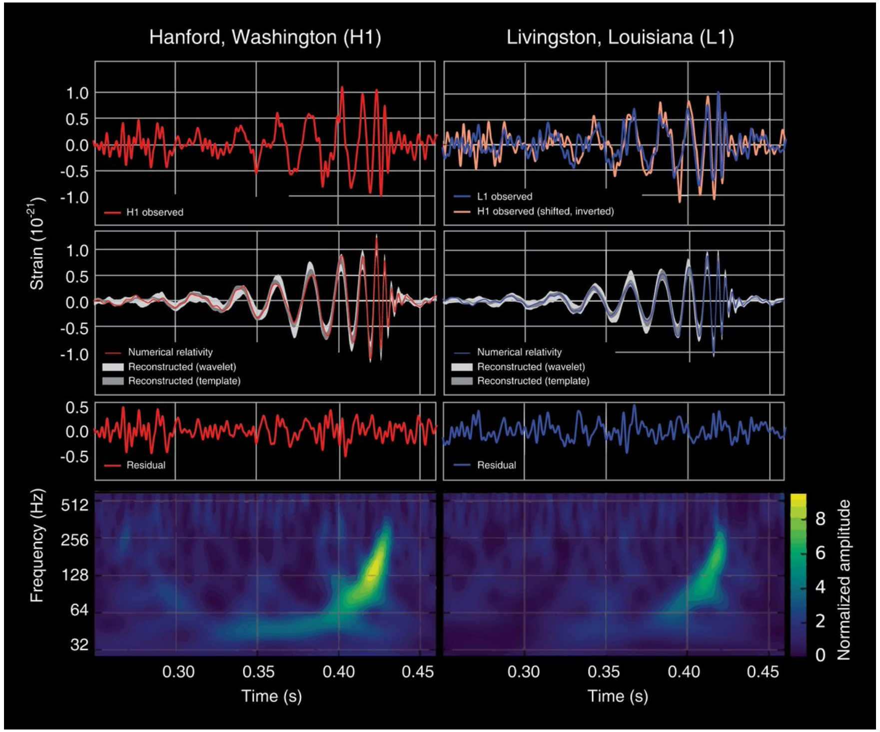
$$W = \lim_{R\to 0} \int_{\infty}^{R} \mathrm{d}r \frac{Gm_1 m_2}{r^2} = \lim_{R\to 0} -\frac{Gm_1 m_2}{R} = -\infty$$

The same is true in classical electrodynamics. The problem of infinite self-energy can be solved in quantum electrodynamics using renormalization techniques.

In gravity we are not that lucky (so far). There are many approaches to resolve gravitational singularities. Today: Can we avoid singularities by modifying General Relativity?

- A simple example: the gravitational potential of a point mass
- Non-local "ghost-free" theories: a promising approach?
- Applications: weak fields, quantum field theory, non-local constitutive law

...but WHY do we need to avoid singularities?



GW150914 (LIGO, September 2015)

M87 (EHT, April 2019)

Compact objects with event horizons exist (both stellar-mass and supermassive)!

Does time and space really end inside of observed objects in our Universe?

A simple example: the gravitational potential of a point mass (1/2)

The gravitational potential of a point-particle is singular at the origin:

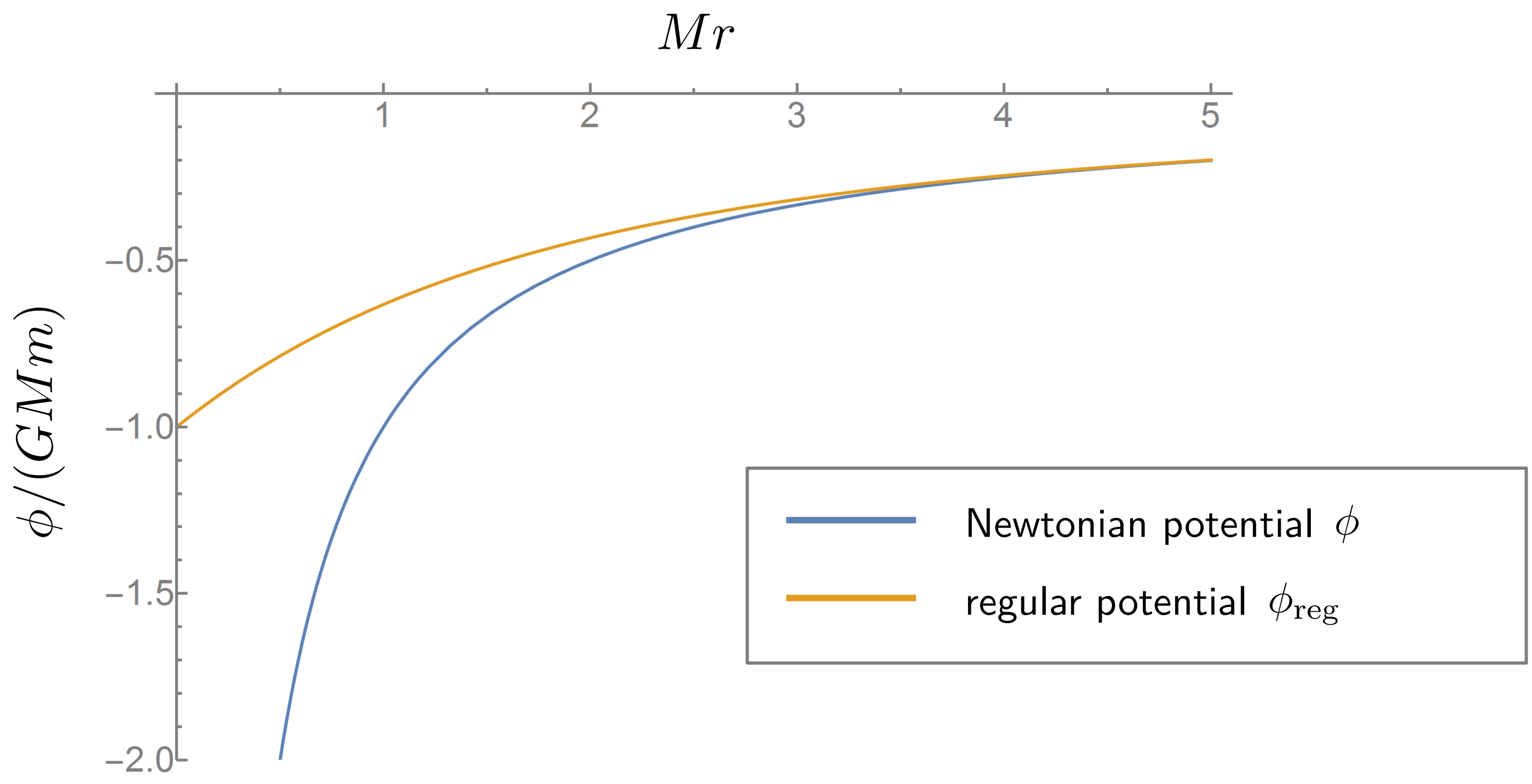
$$\nabla^2 \phi = 4\pi G m \delta(\boldsymbol{r}) \quad \longrightarrow \quad \phi(r) = \frac{-Gm}{r}$$

The pathological behavior at $\boldsymbol{r} = 0$ can be cured by introducing a heavy-mass modification:

$$\nabla^2 (1 - \nabla^2/M^2)\phi_{\mathrm{reg}} = 4\pi G m \delta(\boldsymbol{r}) \quad \longrightarrow \quad \phi_{\mathrm{reg}}(r) = \frac{-Gm}{r}\left(1 - e^{-Mr}\right)$$

This is called Pauli–Villars regularization, and we assume that $M \gg m$ (short distance modification).
For large distances the potential is Newtonian, but for short distance scales it is regularized.

A simple example: the gravitational potential of a point mass (2/2)



Higher-derivative modifications always introduce ghosts at tree-level

The Green function of the Pauli–Villars regularized theory has the following structure:

$$G(\boldsymbol{x}, \boldsymbol{x'}) = \frac{1}{\nabla^2 \left(1 - \nabla^2/M^2\right)} = \frac{1}{\nabla^2} - \frac{1}{\nabla^2 - M^2}$$

The negative sign relative to the original propagator corresponds to a **ghost**. Using this Green function in quantum field theory can lead to negative probabilities and thereby violates unitarity.

$$G(\boldsymbol{x}, \boldsymbol{x'}) = \frac{1}{\nabla^2 \prod_{i=1}^{N} \left(1 - \nabla^2/M_i^2\right)} = \frac{1}{\nabla^2} + \sum_{i=1}^{N} \frac{c_i}{\nabla^2 - M_i^2}, \qquad 1 + \sum_{i=1}^{N} c_i = 0 \,.$$

Generic property of higher-derivative theories. Modification without new poles in propagator?

No more ghosts at tree level!

Modify the Poisson equation by a function $f(\nabla^2/M^2)$ with $f(0) = 1$:

$$\nabla^2 f(\nabla^2/M^2)\phi = 4\pi G m \delta(\boldsymbol{r}) , \qquad G(\boldsymbol{x}, \boldsymbol{x'}) = \frac{1}{\nabla^2 f(\nabla^2/M^2)}$$

If $f \neq 0$ there are no new poles and therefore we can **avoid ghosts**. A possible choice:

$$f = \exp(\nabla^2/M^2) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{\nabla^2}{M^2} \right)^n$$

There are now **infinitely** many derivatives. Some remarks on the equations of motion:

- number of initial data $=$ number of poles in propagator
- initial value problem: Cauchy surface acquires a thickness

The resulting equations are **non-local** in nature. Non-locality can smear out singular quantities.

Effective delta potential

Let us consider a simple one-dimensional example:

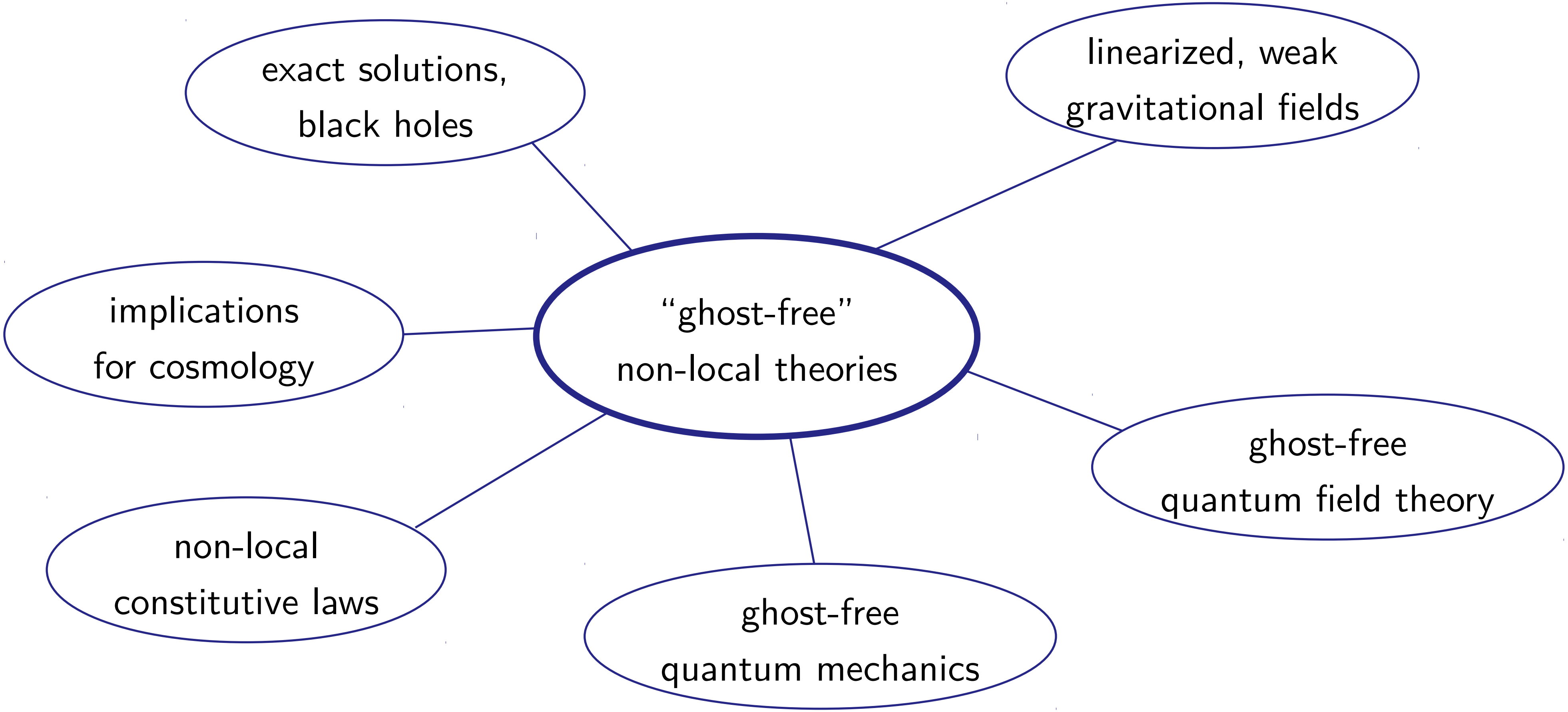
$$\nabla^2\phi = 4\pi G f^{-1}(\nabla^2/M^2) m\delta(r) = 4\pi G \rho_{\text{eff}}(r)$$

A simple calculation gives

$$\rho_{\text{eff}}(r) = f^{-1}(\nabla^2/M^2)\delta(r)$$
$$= \exp(\partial_r^2/M^2) \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}k}{2\pi} e^{ikr} = \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}k}{2\pi} e^{-k^2/M^2+ikr} = \frac{Me^{-(Mr/2)^2}}{\sqrt{4\pi}}\,.$$

Sometimes it is useful to define a **scale of non-locality** $\ell = M^{-1}$ such that $\rho_{\text{eff}}(r) = \dfrac{e^{-r^2/(4\ell^2)}}{\sqrt{4\pi}\ell}$, which is just a Gaussian function (and in the limit $\ell \to 0$ it becomes a delta function).

What has been done? What can we study?



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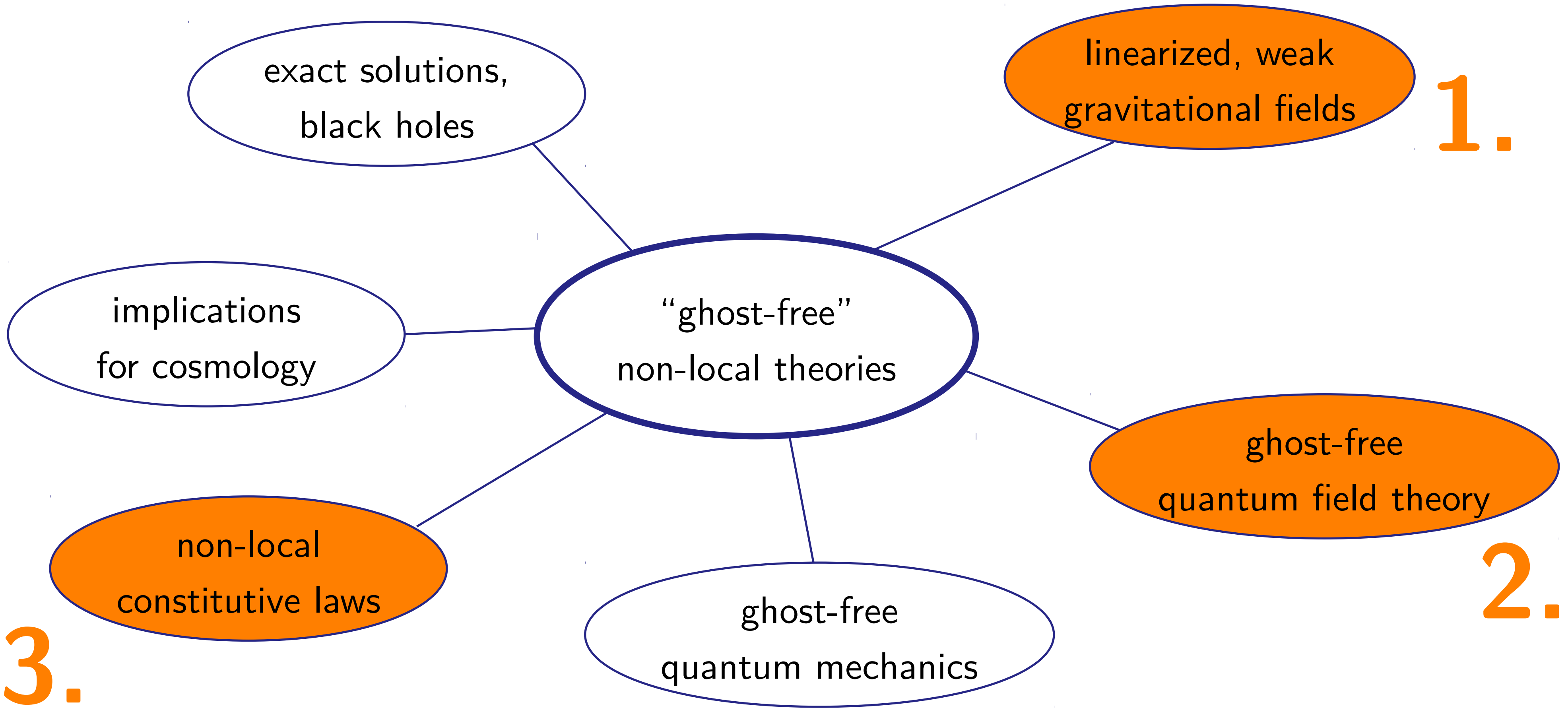


linearized, weak gravitational fields

1.

1. Linearized gravitational potential in ghost-free gravity (1/2)

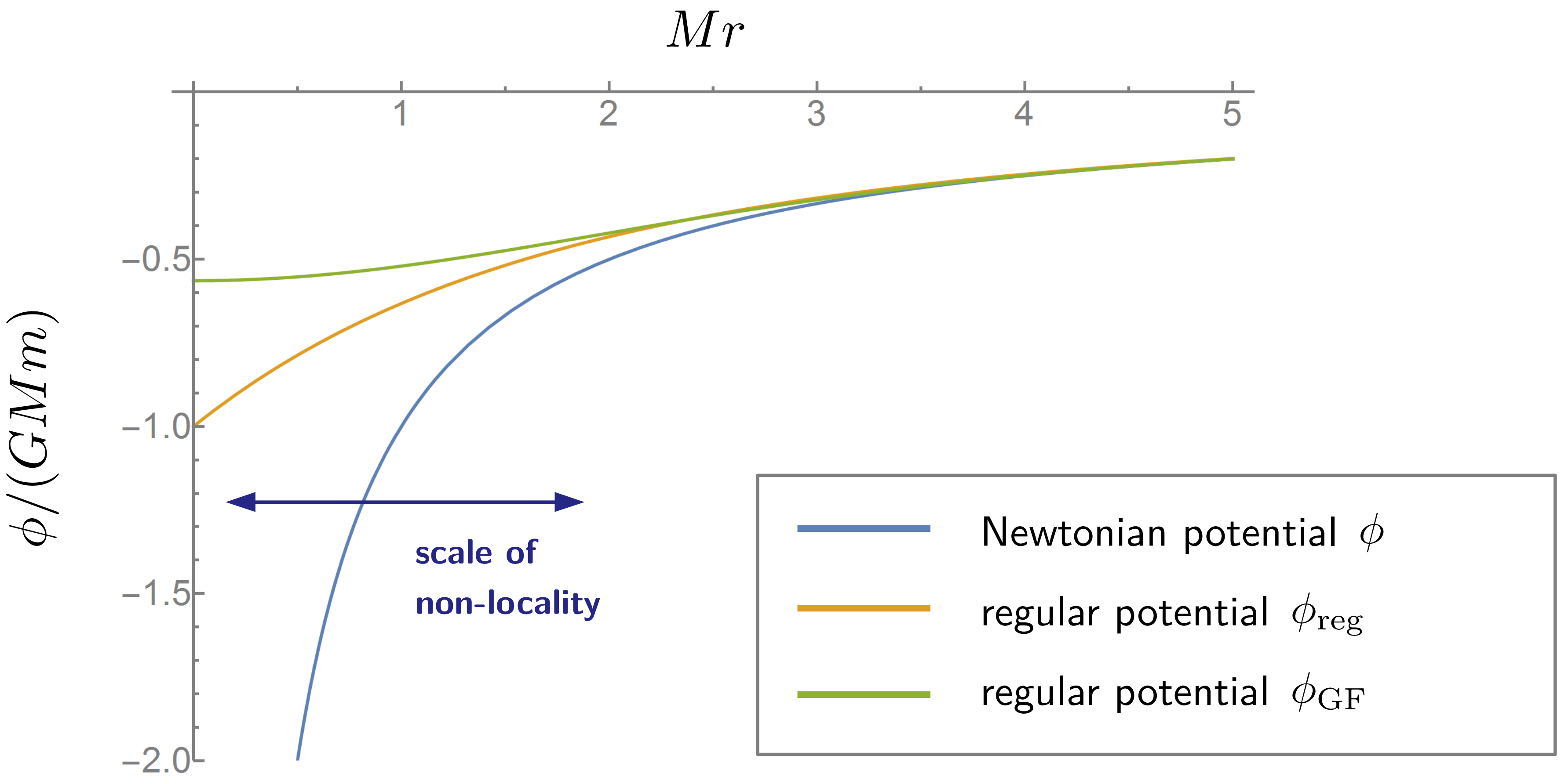
We have shown that in linearized ghost-free gravity one obtains finite potentials as well.

$$S = \frac{1}{2\kappa}\int \mathrm{d}^D x \, \Big(\frac{1}{2} h^{\mu\nu} \, a(\Box)\Box \, h_{\mu\nu} - h^{\mu\nu} \, a(\Box)\partial_\mu \partial_\alpha \, h^\alpha{}_\nu + h^{\mu\nu} \, c(\Box)\partial_\mu\partial_\nu h - \frac{1}{2} h \, c(\Box)\Box h + \frac{1}{2} h^{\mu\nu} \, \frac{a(\Box) - c(\Box)}{\Box} \partial_\mu \partial_\nu \partial_\alpha \partial_\beta \, h^{\alpha\beta} \Big)$$

This corresponds to a non-local Fierz–Pauli action. The functions $a(\Box)$ and $c(\Box)$ are functions of the D'Alembert operator $\Box$. Inserting a static ansatz, perturbing around Minkowski space, one can use Green functions to obtain the regular potential. For $a(\Box) = c(\Box) = \exp(-\Box\ell^2)$ one finds

$$\phi_{\mathrm{GF}}(r) = -\frac{Gm}{r}\mathrm{erf}(Mr/2) \, , \quad M = \ell^{-1} \, .$$

1. Linearized gravitational potential in ghost-free gravity (2/2)



ghost-free
quantum field theory

2.

2. Ghost-free scalar quantum field theory (1/4)

Action: $S[\varphi] = \frac{1}{2}\int \mathrm{d}^d X \left[\varphi f(\Box)\Box\varphi - V(X)\varphi^2\right]\,, \quad f(\Box) = \exp[(-\ell^2\Box)^N]\,, \quad N = 0, 1, 2, \ldots$

For $N = 0$ one arrives at local quantum field theory, but for $N > 0$ there can be new effects related to the **scale of non-locality** ℓ . We expect that these effects vanish in the local limit $\ell \to 0$.

Field equations: $f(\Box)\Box\varphi = V(X)\varphi$

For a delta-shaped potential, $V(x) = \lambda\delta(x)$, these can be exactly solved using the Lippmann-Schwinger method. For simplicity, we will consider the static case in two dimensions.

$$G(X, X') = -\int\limits_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega(t-t')} \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}k}{2\pi} e^{+ik(x-x')} \frac{e^{-[\ell^2(k^2-\omega^2)]^N}}{\omega^2 - k^2}$$

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analytic properties?

2. Ghost-free scalar quantum field theory (2/4)

$$G^{\text{loc}}(X, X') = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega(t-t')} \int_{-\infty}^{\infty} \frac{\mathrm{d}k}{2\pi} e^{+ik(x-x')} \times \begin{cases} -\dfrac{1}{(\omega - i\epsilon)^2 - k^2} & \text{retarded} \\[2ex] -\dfrac{1}{(\omega + i\epsilon)^2 - k^2} & \text{advanced} \\[2ex] -\dfrac{1}{\omega^2 - k^2 + i\epsilon} & \text{Feynman} \end{cases}$$

The analytic properties of the free local propagator in Fourier space dictate the causal properties of the free local Green function in real space. The same is true in **non-local** theories!

2. Ghost-free scalar quantum field theory (3/4)

Let us write the total Green function in Fourier space as a sum of two terms:

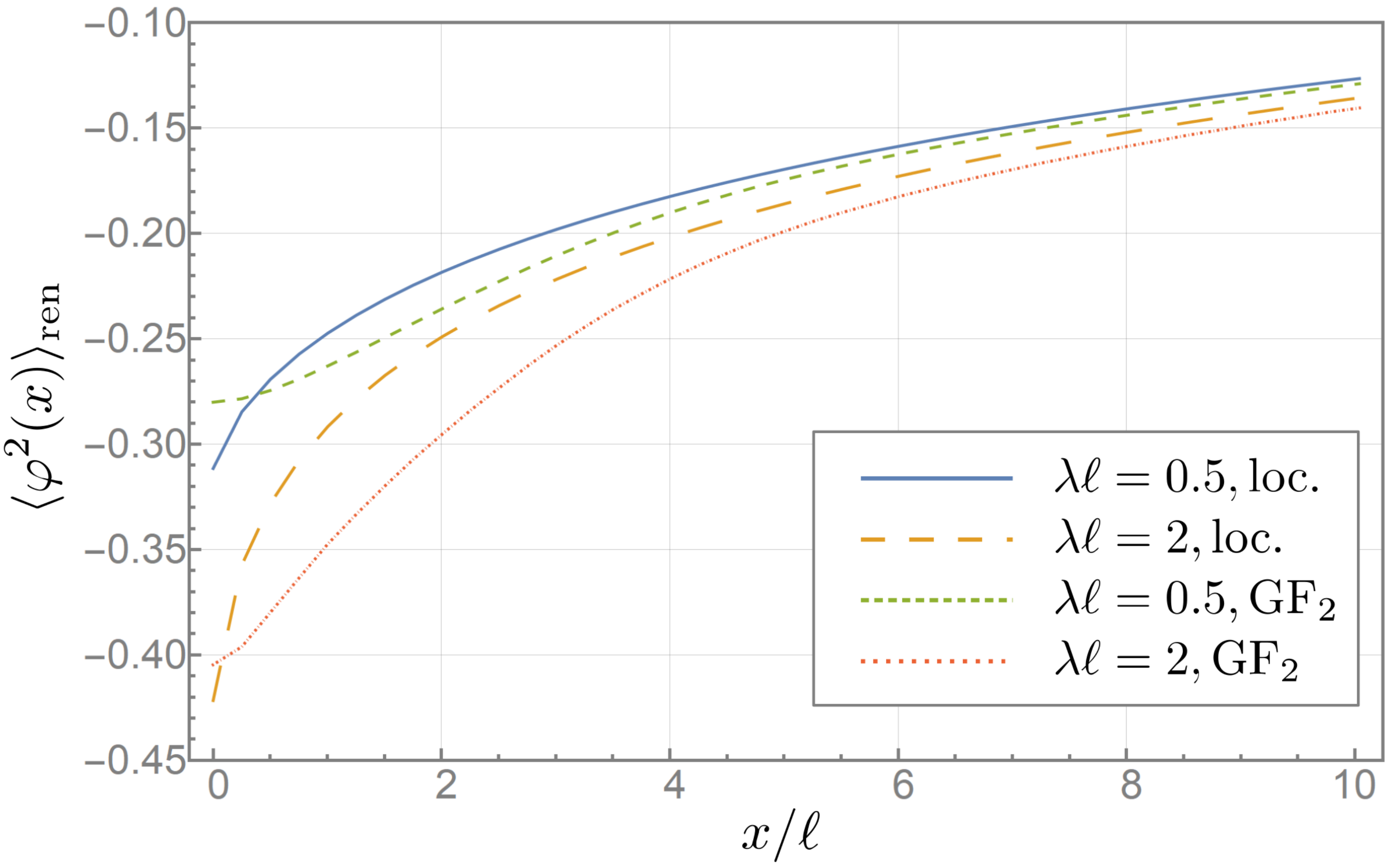
$$G_{\omega,k} = G^{\text{loc}}_{\omega,k} + \Delta G_{\omega,k} = \boxed{\frac{1}{k^2 - \omega^2}} + \boxed{\frac{e^{-\ell^{2N}(k^2-\omega^2)^N} - 1}{k^2 - \omega^2}}, \quad N = 0, 1, 2, \ldots$$

Let us treat the **analytic properties** separately. The **non-local modification** is completely regular at $\omega = k$ and behaves like $\sim -\ell^{2N}(k^2 - \omega^2)^{N-1}$. Same analytic properties as in local case!

The behavior in the frequency domain is convergent for even N and divergent for odd N.

In quantum field theory, only even N makes sense. In quantum mechanics, both even and odd N work well, since no summation over frequencies is performed.

2. Ghost-free scalar quantum field theory (4/4)



non-local constitutive laws

3.

3. Non-local constitutive law

In classical electrodynamics, non-locality can enter the theory via the constitutive law that relates the field strength 2-form $F_{\mu\nu}$ to the excitation/response (n-2)-form $H_{\mu\nu}$.

$$H^{\mu\nu} = \frac{1}{2}\chi^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

replace with non-local integral kernel.

A similar procedure can be applied to the torsion tensor $T_{\mu\nu}{}^{\rho}$ in the teleparallel equivalent of General Relativity (non-local gravity model of Mashhoon & Hehl, 2008).

Giving up Einstein's clock hypothesis induces non-locality in their model. Note that any acceleration gives rise to a distance scale. $g = 9.81\,\mathrm{m/s^2}$ corresponds to $d = c^2/g \sim 10^{16}\,\mathrm{m}$.

Non-locality at very large scales. Can predict dark matter. How is this model related to the ghost-free gravity model? Can we use calculational techniques from one model to learn more in the other?

Conclusions and outlook

The field of ghost-free physics is very interesting and has many open problems. Instead of focusing on conceptual issues, we find it insightful to study concrete problems, such as

- the weak-field limit of ghost-free gravity
- quantum-mechanical scattering problem
- vacuum polarization in quantum field theory
- Hawking radiation

It would be interesting to extend these studies into the strong field regime in proximity of black holes.

Thank you for your attention.

Abstract

Singularities are a well known problem of Einstein's General Theory of Relativity. It is believed that any consistent theory of gravity should resolve them. In this talk, we will explore such an avenue in the context of classical non-local gravity.

In particular, I will present the model of so-called “ghost-free gravity” (at the linearized level). This can be thought of as a generalization of Fierz–Pauli theory with infinitely many derivatives. These derivatives, unlike in Pauli–Villars regularization schemes, are combined in such a way that there appear no unphysical ghost modes in the propagator at tree level. I will demonstrate that the Newtonian potential is regularized in this theory, and comment on steps towards exploring ghost-free gravity in the strong field regime of black holes.

If time permits I will also comment on non-local gravity formulated via a non-local constitutive law. At the linear level, this formalism is closely related to ghost-free gravity, even though the origin of non-locality might be of an entirely different nature.