Report on the symposium of June 17, 2021

The "PhD/Early Postdoc Symposium on Non-locality" is a strategic long-term effort to foster communication and exchange of ideas between early-career scientists. You can find out more about us on our website www.spintwo.net/Symposium/. This is the report on the first symposium, which took place on June 17, 2021.

Suggested program

1. Identification of important unsolved problems

- QFT: clear statements and status on unitarity
- QFT: role of Wick rotation and Sen prescription
- QFT: gauge theories and determining Feynman rules
- QFT: renormalizability
- GR: status of exact solutions (black holes, cosmology, ...)
- GR: can regular black hole solutions be found?
- general: where does the form factor come from?
- general: observational tests for non-locality?

2. Exploration of possible connections to other fields

- astrophysical objects and their interaction with non-locality (stellar objects, gravitational waves, ...)
- non-locality in quantum effective actions
- non-locality in quantum entanglement and quantum information
- diffeomorphism invariance in quantum gravity requires non-locality
- role of holography

3. Future plans

- Fixing a time for the next symposium
- Let us build something together and develop future perspectives.
- Long-term goal: Reviews of Modern Physics or even Physics Reports review article?

Attending participants

- Ali Akil (PhD student, Southern University of Science and Technology, China)
- Jens Boos (postdoc, William & Mary, United States)
- Luca Buoninfante (postdoc, Tokyo Institute of Technology, Japan)
- Anish Ghoshal (postdoc, INFN Tor Vergata Rome, Italy)
- Breno Loureiro Giacchini (postdoc, Southern University of Science and Technology, China)
- Ivan Kolar (postdoc, Van Swinderen Institute, University of Groningen, Netherlands)
- Sravan Kumar (postdoc, Tokyo Institute of Technology, Japan)
- Yuichi Miyashita (master student, Tokyo Institute of Technology, Japan)

- Desmond Villalba (visiting assistant professor, Drury University, United States)

Jens opened the meeting and briefly gave an introduction to the PhD/Early Postdoc Symposium on Non-locality. It is supposed to be a safe space for discussions between early-career researchers, where anything and everything can be discussed. In particular, the question "I don't think I understood this, could you explain it again?" is highly encouraged. After a brief icebreaker the first main discussion session was opened.

1 Important unsolved problems in non-local physics

1.1 Status of exact solutions

Anish mentioned that besides exact solutions in General Relativity it is also very difficult to find exact solutions in Yang–Mills theory or even in ϕ^4-theory. Luca agreed and Sravan added that it is a fundamental problem how to solve the classical equations of motion, with Witten *et al.* having found some way to solve these equations [1]. Luca added that even for a scalar field in one dimension in ϕ^4-theory one of the very few exact solutions is the instanton tanh-solution which connects two vacua. It is not clear how to approach this in the non-local setting.

Jens mentioned that the approach to solving the equations depends a lot on the theory, each one coming with different perturbative methods. Naively expanding non-local form factors such as $\exp(\ell^2 p^2)$ in powers of ℓ is very counter-intuitive and not helpful in general. It is even worse in gravity where the $\Box$-operator contains the field itself as well (i.e. the metric), literally making this infinitely complicated. Recent progress has been made by Ivan in non-local gravitational theory [2], by Luca and Yuichi with scalar lumps [3], and also Anish in the context of non-local gauge theories [4]. Sravan mentioned that in cosmology huge simplifications can be done, and we would cycle back to this discussion later below.

Anish mentioned that even in the local theory a fundamental question is till what order can we trust perturbation theory? Witten stated [Q: reference?] that perturbation theory is valid until instanton solutions starts to dominate. As an open question, Jens asked whether non-locality will make this transition easier or not.

Jens mentioned that in cosmology there is a simplifying assumption, $\Box R = aR + b$, that can be imposed on scalar curvature that renders many non-local to a much simpler form. Sravan responded that such $\Box R$-eigenvalue condition comes from the trace equation of the local theory, and is indeed an exact solution in non-local theories. With matter, however, the situation becomes much more complicated. Jens wondered whether one throws away solutions when assuming the $\Box R$-eigenvalue equation. Sravan responded that in order to go beyond this one would need to consider orthogonal eigenmodes of the $\Box$-operator to be able to perform a Fourier-style decomposition in curved spacetime. This could be used to check and see if there are any other solutions beyond the ones already found, upon which Luca added that all of these considerations take place in vacuum (read: no matter).

Anish asked whether any of those statements are gauge-dependent, which Sravan denied. Ivan asked if this only works for gravitational Lagrangians of the form $Rf(\Box)R$ or if one may go beyond that. Sravan and Luca responded that one can have a Weyl term, too, which vanishes in the cosmological case due to conformal invariance. A non-zero Ricci tensor makes it more complicated, but is not so important because the interest in cosmology is mainly in expansions around de Sitter space. But it is non-trivial to generalize this to other scenarios, e.g. black hole backgrounds with non-zero Weyl. The Fourier decomposition of a scalar is understood, but the Fourier transformation of tensorial objects is more difficult.

Ivan asked to confirm that the statement that the eigenfunctions of the $\Box$-operator form a complete set is a rather strong assumption, to which Sravan replied that in cosmological scenarios, as long as the Hubble parameter has no singularities it is justified (the $\Box$-operator contains the Hubble parameter). Without that assumption, however, it basically becomes a question whether these spacetimes are physical or not, but certainly it is an open question for a general geometry. Jens asked about the role of global hyperbolicity, since in black hole spacetimes some metric coefficients become zero. Sravan responded that in cosmology this is not really a problem because the scale factor is always positive.

Sravan then mentioned that if one is interested in black hole solutions in modified gravity one might look into Stelle gravity (i.e. Langrangians that contain quadratic expressions in the curvature) which can be thought of as "a bit" beyond General Relativity. As Stelle himself mentioned during the recent conference [5], obtaining exact black hole solutions is a huge problem in those theories, and can typically only be done numerically [6, 7]. Ivan mentioned that in his experience, the non-linearity is the hardest part. It is possible to look for solutions with specific symmetries/specific algebraic curvature types (which is used in quadratic gravity). Another option is to look an ansatz that yields effectively linear field equations, a group of which Ivan recently described and called "almost universal class" of geometries [2].

Sravan shared that in his opinion it is "always hopeless" to find exact analytic solutions in complicated non-linear theories. One interesting question could be whether non-locality could give deviations, order by order, from exact symmetries? Then the question was raised how one would impose a certain symmetry (say, spherical symmetry) on a metric ansatz. Jens and Ivan mentioned that typically one demands the Lie derivative to vanish along some directions (the Killing vectors that generate the isometry group), but that would result in a differential condition on the curvature, rather than an algebraic relation as in the case of conformal invariance where the Weyl tensor vanishes identically. Sravan mentioned that Breno recently worked on spherically symmetric and regular solutions [8], upon which Breno commented that indeed there has been some success, but strictly in the linear regime, beyond which not much is known. In a concluding section, Ivan summarized that there exist no closed-form algorithms for non-linear cases. The best one can hope for is to find a good ansatz on, say, the curvature, which reduces the field equations to be of either a local and non-linear form or to be of non-local and linear form. For equations that are both non-local and non-linear, which is probably always the case for spherically symmetric solutions, we have no idea how to solve them.

1.2 Non-linearity vs. non-locality

Jens mentioned that the competition of non-locality and non-linearity can be understood from a simple example given to him by Valeri Frolov. The curvature of a Schwarzschild black hole goes like $R \sim m/r^3$. Squaring results in m^2/r^6, whereas differentiating twice gives m/r^5. It is clear that non-linearity diverges faster than non-locality (read: higher derivatives). Anish asked whether this was a general principle, upon Jens mentioned that the only requirement seems to be the existence of a dimensionful parameter (such as the mass parameter). Jens then asked Luca if he would agree that this observation by Frolov is similar to the competition one encounters in quantum field theory formulated in Euclidean space, where at the level of momentum-space Feynman rules there is always a competition between propagators that typically decrease with momentum and vertices that typically grow with momentum. Vertices represent non-linearity, and propagators represent non-locality.

Luca responded that in non-local gravity one indeed has features both in the propagator and the vertex. In the Euclidean setting the propagator is suppressed and the vertex is enhanced. One can perform a power-counting renormalizability study and finds for v vertices and p propagators a superficial degree of divergence D of $v - p$. Using the standard topological loop argument this becomes that $D = 1 - l$ where l is the number of loops, which implies that for more than 1-loop everything is suppressed. At 1-loop one needs to check explicitly, but this is expected since D is the superficial degree of divergence.

At the non-local conference [5] Terry Tomboulis mentioned that in R^2 and Ric^2 theories the tree-level amplitude looks divergent, whereupon Modesto mentioned that this theory with external on-shell massless gravitons coincides with Einstein gravity. Nobody at the symposium disagreed with Modesto's conclusions. If one, however, includes a Riemann^2-term then the divergence problem can still be there at tree-level because the on-shell condition of $R_{\mu\nu} = 0$ leaves the Weyl part of the curvature unspecified. Sravan emphasized that of course the role of the Lagrangian is very important, and in his comments Modesto may have missed that part. Sravan continued that there are two ways to construct Lagrangians of that type: either with a scalaron or without one, where one must have a scalaron from the early universe perspective. And even if one does not include it to begin with, 1-loop divergences will always bring R^2-terms which will bring a scalaron.

Luca encouraged the participants to perhaps discuss other points as well, and Anish mentioned that one should face similar problems in simpler theories as well, such as a cubic scalar theory, wherein vertices compete with propagators as well. These theories arise after symmetry breaking in $\lambda\phi^4$ theories, and these theories have been studied recently e.g. in [9]. Sravan asked why one would expect non-locality to play a role at the electroweak scale, i.e. the scale of spontaneous symmetry breaking. There was some confusion on terminology, after which it became apparent that some people use the word "Higgs" more general for theories with spontaneous symmetry breaking, whereas others use the word "Higgs" solely for the particle in the standard model.

1.3 Non-Abelian non-local gauge theories

Luca mentioned in this context that one of the points in the program referred to Feynman rules in non-local non-Abelian gauge theories and wondered what the problem with that was. Jens responded in a clarification that determining Feynman rules in a non-Abelian non-local gauge theory can be quite difficult because the covariant derivative appears in the exponential. This means that there are a lot of different vertices, in particular those with two scalars and an arbitrary amount of gauge bosons. The paper by Tomboulis [10] needs these vertices for the study of power-counting renormalizability, and the combinatorics are quite tricky. Luca mentioned that the gauge invariance can be checked perturbatively order by order, so it should not be an unsolvable problem, but Jens pointed out that the Tomboulis paper never was published. It was mentioned that the reason for that was not the power-counting renormalizability argument, but rather some issues with the attempted Kjallen–Lehman representation for non-local propagators that prompted string theorists to reject that part of the paper. A PhD student of Tomboulis at UCLA, Marcus Eran (committee members: Zvi Bern and Sergio Ferrara) was devoted to the study of all such vertices for gauge theories. The document is not publicly available but can be supplied to anybody who is interested. In that context there recently appeared a paper by Modesto, Piva, and Rachwal on non-local gauge theories [11] where it is conceivable that Piva used results from this PhD thesis.

After that, the discussion moved on to the second main topic:

2 Connections to other fields

2.1 Wavelet approach

Sravan mentioned, à propos Tomboulis, that Tomboulis recently had a paper on wavelet methods [12] that he also mentioned during his talk at the recent conference [5] in March. There was some sort of confusion as to what he is writing about, since there is an application to non-locality using wavelets. It was suggested to devote a future symposium to a discussion of these methods.

2.2 Diffeomorphism invariance and non-locality

Luca asked about one of the points in the program mentioning that "diffeomorphism invariance requires non-locality." Jens pointed out recent work by Donnelly and Giddings [13], wherein they construct manifestly diffeomorphism-invariant observables that are thought to become necessary in any theory of quantum gravity. These observables, defined at a point x, are typically non-local since they involve integrations of fields around that point as well.

For example, in a simpler setting of U(1) theory these gauge-invariant observables correspond to so-called Wilson line dressings. In gravity a similar construction has been performed order by order [13]. These are invariant under gauge transformations that vanish at infinity, and they admit an operational understanding related to the preparation of the quantum state they describe. In the case of electrodynamics it means that one cannot just consider the gauge-dependent matter field itself, one necessarily has to consider its associated photon along with it, which is then gauge-independent. This is explained in Ref. [13] in Eqs. (4) and (5). These are non-local equations since they require knowledge of the field far away, and can also be generalized to methods proposed by Dirac [Q: does anybody have the reference?], and the methodology appears somewhat similar to smearing functions in non-local theories.

2.3 Quantum non-locality and entanglement

Ali asked whether there is any reason to believe that the non-locality encountered in quantum entanglement and quantum information has anything to do with the non-locality in non-local gravity. Jens mentioned that, similar to the "Higgs" confusion earlier, "non-locality" can mean very different things in different fields. Luca added that the non-locality in infinite-derivative theories is totally different from quantum non-locality, which is a principle that follows from local equations of motion and has nothing to do with violation of causality. Infinite-derivative theories, on the other hand, really modify dynamics. Jens mentioned that the two non-localities may not be completely independent since interaction leads to the entanglement of systems, and non-locality in infinite-derivative theories smears out localized interactions. Luca added that of course non-locality can affect quantum mechanics, but non-locality of entanglement exists independently from that. For example, via the Bell experiment one can argue that quantum entanglement is just a manifestation of how quantum probabilities work, and that is not related to dynamics. Desmond commented that the discussion on the foundations of quantum mechanics is also related to the school of thought of quantum mechanics, and pondered whether for example the well-known EPR paradox could be an interesting avenue to pursue that involves modifying our philosophy.

2.4 Quantum effective actions

Yuichi commented that quantum effective actions contain terms such as $\log(p^2)$, which are non-local, but that it is not clear whether these are just calculational techniques or necessarily related to new physics. Jens mentioned that effective actions in QFT typically only contain dimensionful parameters of the original action, which is a notable difference to infinite-derivative non-local theories which need a dimensionful scale of non-locality (called ℓ or M_s). Moreover, our standard model of particle physics, for all we know, could also just be an effective action. Sravan wondered whether there could be observational constraints of $\log(\Box)$ in gravity, particular in the infrared, since that is where this term becomes relevant. Anish asked to clarify whether Sravan was talking about Lagrangians containing $R\log(\Box)R$, which he confirmed. Sravan wondered about constraints from dark energy, and mentioned the Deser–Woodard $R1/\Box R$ non-local model [14], as well as models proposed by Maggiore [15]. Jens added that Mazumdar has also studied IR-modified gravity with a $\exp(-M_s^2/\Box)$ form factor [16].

2.5 Emergent non-locality

Coming back to an earlier comment by Anish on work by Dvali, who mentioned at the recent conference [5] that it is possible to obtain non-locality by integrating out auxiliary fields, Luca asked whether there is any simple toy model. Jens mentioned that he just worked on a paper that does exactly that [17].

3 Closing remarks

In the closing part of the symposium, the participants expressed the wish to turn the symposioum into a recurring event, with the next date to be fixed later. Jens also briefly commented on possible future directions of the symposium, emphasizing the long-term idea of creating a community of graduate students and postdoctoral researchers working in the field of non-locality. A possible long-term project could be the writing of a review article on non-locality for Reviews of Modern Physics or even Physics Reports, if we can find some senior person to invite us to write such a review.

After two hours of ample discussion and exchange, Jens closed the meeting and thanked all participants for their time and interest. Due to the "Quantum gravity and all of that" seminar series [18], which was scheduled right after the symposium, many of the participants saw each other immediately in that seminar.

Acknowledgements

We thank William & Mary for the kind provision of Zoom services, and the National Science Foundation for financial support under grant PHY-1819575.

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