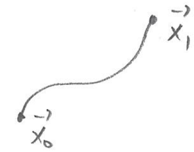
Score: topological phases 7 diff forms in gauge theory 4
BH spacetimes 5 curvature & torsion 4

12. Topological phases



Wilson line: $W = e^{-ie\int_{\vec{x}_0}^{\vec{x}_1} \vec{A}\cdot d\vec{x}} \, \Psi(\vec{x}_1)$

gauge trafo: $\vec{A} \to \vec{A} + \vec{\nabla}\lambda$, $\Psi \to e^{ie\lambda(x_1)}\Psi(x_1)$

$$W \to e^{-ie\int_{x_0}^{x_1} \vec{A}\cdot d\vec{x} - ie\cancel{\lambda(\vec{x}_1)} + ie\lambda(\vec{x}_0)} \, \cancel{e^{i\lambda(\vec{x}_1)}} \, \Psi(x_1)$$

$$= e^{ie\lambda(\vec{x}_0)} \, W(x_0, x_1) \quad \text{gauge-covariant.}$$

Example: Ψ = wave function of an electron / charged particle

Closing a Wilson line: $\oint_{\mathcal{C}} \vec{A}\cdot d\vec{x}$ gauge-invariant itself
e.g. bring an electron in a closed loop.

Differential forms: p-forms can be integrated over p-dimensional volumes

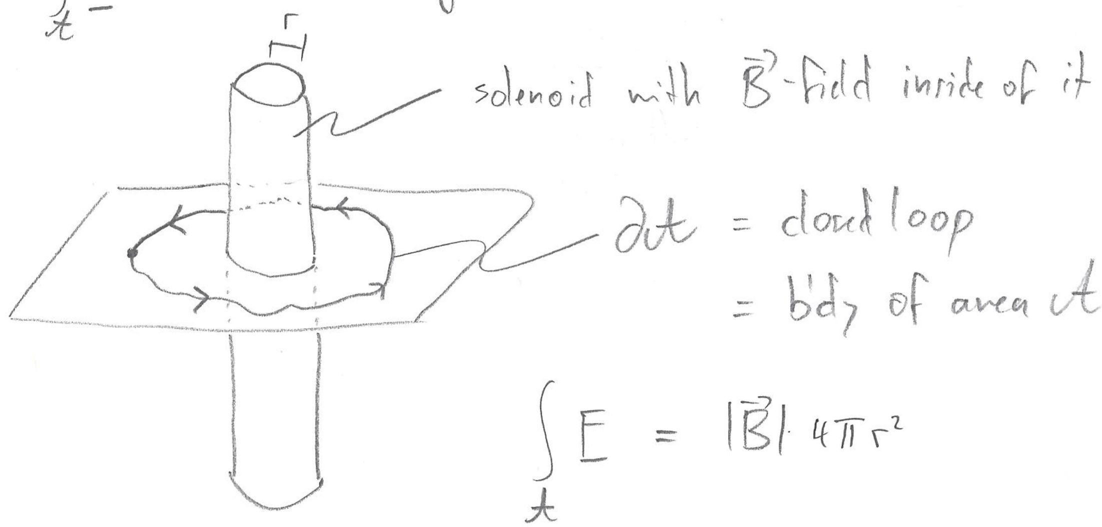
$$\int_{\mathbb{R}^p} \underline{\omega} = \int_{\mathbb{R}^p} dx^1 dx^2 \ldots dx^p \, \omega_{12\ldots p}$$

$p=1$: $\int \underline{A}$, closed loop: $\oint_{\mathcal{C}} \underline{A}$

Stokes' theorem: $\oint_{\partial\mathcal{A}} \underline{A} = \int_{\mathcal{A}} d\underline{A} = \int_{\mathcal{A}} \underline{F}$,

where $\underline{F} = d\underline{A}$, $\mathcal{A}$ is an area, and $\partial\mathcal{A}$ is its boundary

What is $\int_{\mathcal{A}} \underline{F}$? Flux through the surface!



$$\int_{\mathcal{A}} \underline{F} = |\vec{B}| \cdot 4\pi r^2$$

Example: charge quantization

$$ie \int \vec{A} \cdot d\vec{x} = ie \frac{1}{2\pi r} \cdot 2\pi r = ie = i \mathbb{Z} \hbar$$

quantization of charge!

General structure of field strength tensors:

$$\underline{F} = \frac{1}{2} \underbrace{F_{\mu\nu}}_{\text{2-form indices}} \underbrace{{}^{A}{}_{B}}_{\text{Lie algebra indices}} dx^\mu \wedge dx^\nu \underbrace{t_A{}^B}_{\text{generator of Lie algebra}}$$

$$\int_{\mathcal{A}} \underline{F} = \text{group element}$$

examples:

$$\oint A \hat{=} \text{charge}$$

$$\int \frac{1}{2} R_{\alpha\beta}{}^{\mu}{}_{\nu} L_{\mu}{}^{\nu} dx^\alpha \wedge dx^\beta = \text{Lorentz transformation}$$

$$\int \frac{1}{2} T_{\alpha\beta}{}^{\mu} \nabla_\mu dx^\alpha \wedge dx^\beta = \text{translation}$$