Superradiance of rotating black holes as a probe for light bosons

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Introduction

Black holes (BHs) have long been the subject of theoretical studies of General Relativity, but with the detection of gravitational waves [1] this picture has changed: the statistically significant signal GW150914 constitutes just as much a *direct detection of black holes* as the 125 GeV cross section excess did in 2012 for the first detection of the Higgs particle [2].

Why is this important? For a long time, BHs were studied as exact, but abstract solutions of the Einstein field equations, and there was no experimental confirmation of whether these objects are indeed realized in Nature. This is no longer the case: GW150914 has shown astrophysical BHs to exist. Within a few years, the Event Horizon Telescope Collaboration [3] will be able to provide detailed images of the center of our galaxy, providing data on the supermassive BH in Sagittarius A.

We see: seemingly entirely theoretical concepts, such as stability analyses of BHs, suddenly become actual physical processes that might even have observational significance. To that end, in this review we would like to understand how the mechanism of *superradiance* of rotating BHs can provide signals that lead to the detection of very light bosons (we fill focus on axion-like particles). In this review, we would like to understand the work done by Arvanitaki *et al.* [16, 17, 18] in that regards. We proceed as follows:

In Sec. 1, we briefly review the history of superradiance (SR) and emphasize that it is a kinematic effect occurring not only around BHs, but more generally in dissipative media. We will briefly discuss the anomalous Doppler effect in the Cherenkov radiation of a moving particle, as well as the Klein paradox as realizations of SR. We finish this section by reviewing rotational SR and deriving a condition that—if satisfied—determines the onset of SR for a scalar field around a Kerr BH. Sec. 2 is devoted to the discussion of light bosons: we briefly explain the necessity for the axion in context of the strong CP problem of quantum chromodynamics, and explain why its mass has to be small. Then, in Sec. 3, we combine these concepts: we describe the "gravitational atom," that is, the distribution of light bosons around a massive object. Moving on to the dynamics of superradiant processes, we discuss the relevance of different mechanisms that may all give rise to distinct observable consequences that will allow the observer to detect a light boson, or at least put strong constraints on its mass. Finally, in Sec. 4, we review various observational signatures from the processes discussed in Sec. 3, and conclude the review by sketching extensions of the model presented here.

1 Superradiance

In this section, we will briefly review several examples of SR. For a more detailed overview of general superradiant phenomena see Bekenstein and Schiffer [4], and for an extensive introduction see Brito *et al.* [5].

The first notion of SR occured in 1947, when Ginzburg and Frank [6] noted that an electrically neutral particle (in its ground state) moving uniformly through a medium may emit a photon. This is unexpected, since in vacuum the simultaneous constraints of conservation of energy and momentum as well as Lorentz invariance prohibit such a process. The *anomalous Doppler effect* is caused by such a mechanism: suppose a system moves through a medium with a frequency-dependent index of refraction $n(\omega)$ such that the emitted radiation from that particle in the rest frame of the medium is [7]

$$\omega(\theta) = \frac{\omega_0\sqrt{1-v^2}}{1 - vn(\omega)\cos\theta}. \tag{1}$$

Here, v is the linear velocity of the particle, θ denotes the angle of observation, and ω_0 is the frequency of emitted radiation in the particle's rest frame. Suppose now that the particle is moving superluminally, that is, $vn > 1$. Then the denominator can become negative. If the left-hand side is supposed to be positive (observed radiation frequency), then it implies that $\omega_0 < 0$, in turn implying that the system loses more kinetic energy than the emitted photon carries off. Hence, the internal energy of the system must increase.

Another example is the so-called *Klein paradox* [5]: consider the Klein–Gordon equation for a charged scalar field moving towards a strong potential. Then, it can be shown, the interaction with the potential gives rise to the production of pairs of particles. Specifically, the amplitudes $\mathcal{I}$, $\mathcal{T}$, and $\mathcal{R}$ of the ingoing, transmitted, and reflected waves, respectively, can be shown to be related via

$$|\mathcal{R}|^2 = |\mathcal{I}|^2 - \frac{\omega_0 - eV}{\omega_0}|\mathcal{T}|^2 \tag{2}$$

Observe that for $0 < \omega_0 < eV$ one has $|\mathcal{R}| > |\mathcal{I}|$, which amounts to an amplification of the reflected current, also known as superradiance. A generalization to fermions does not yield the same result. Employing the Dirac equation instead, one finds no amplification:

$$|\mathcal{R}|^2 = |\mathcal{I}|^2 - |\mathcal{T}|^2 . \tag{3}$$

The reason for these relations comes from the properties of scalar and fermionic matter currents, but it can also be understood intuitively: fermions obey the Pauli exclusion principle, and thus one cannot have amplification of fermions in a single mode.

Another example was found by Zel'dovich in 1971 [8]: An absorbing cylinder, rotating with a fixed angular velocity of Ω, will no longer absorb an incoming wave with a phase velocity ω/m provided

$$\Omega > \frac{\omega}{m}, \tag{4}$$

where m denotes the azimuthal angular eigenvalue. Zel'dovich conjectured immediately that the same should hold for a rotating black hole, where the absorbing surface is replaced by the event horizon (much in the spirit of the membrane paradigm [7]). Independently, Misner [9] claimed that Kerr black holes may amplify incident waves.

1.1 Superradiance in the Kerr spacetime

Let us now address the situation for a Kerr black hole [10]. In Boyer–Lindquist coordinates, the metric can be written as

$$\begin{aligned} \mathrm{d}s^2 &= -\left(1-\frac{2Mr}{\Sigma}\right)\mathrm{d}t^2 - \frac{4Mra\sin^2\theta}{\Sigma}\mathrm{d}t\,\mathrm{d}\phi + \frac{A\sin^2\theta}{\Sigma}\mathrm{d}\phi^2 + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma\mathrm{d}\theta^2, \\ \Sigma &:= r^2 + a^2\cos^2\theta, \quad \Delta := r^2 - 2Mr + a^2, \quad A := \left(r^2+a^2\right)^2 - \Delta a^2\sin^2\theta. \end{aligned} \tag{5}$$

It admits the timelike Killing vector $\xi_{(t)} := \partial_t$ as well as the angular Killing vector $\xi_{(\phi)} = \partial_\phi$. Let us define an observer moving along $(r,\theta) = \text{const.}$ trajectories, such that their 4-velocity u is proportional to $\eta = \xi_{(t)} + \omega\xi_{(\phi)}$ according to $u = \eta/\sqrt{|\eta^2|}$. For the interval $\omega_- < \omega < \omega_+$ this vector is timelike, but for $\omega = \omega_\pm$ it becomes null. Outside the BH ergosphere, $r > r_e = M + \sqrt{M^2 - a^2\cos^2\theta}$, ω_i is negative. For $r = r_e$, however, $\omega_- = 0$ and the observer has to corotate. At the horizon $r_h = M + \sqrt{M^2-a^2}$, the two values $\omega_\pm$ coincide into the value we shall call the *angular velocity of the BH*:

$$\Omega \;:=\; \omega_\pm(r_h) \;=\; \frac{a}{2Mr_h} \;=\; \frac{a}{r_h^2+a^2} \;=\; \frac{a}{2M^2+2M\sqrt{M^2-a^2}} = \frac{1}{2M}\left(\frac{J}{M^2+\sqrt{M^4-J^2}}\right), \tag{6}$$

where in the last line we have reinstated the angular momentum of the black hole $J := aM$. At the horizon, the vector field $\ell := \xi_{(t)} + \Omega\xi_{(\phi)}$ corresponds to the null generator of the horizon.

Based on thermodynamical considerations, the Hawking area theorem [7] states that the surface area of a BH can never decrease. The surface area of the Kerr black hole is

$$\mathcal{A} = 8\pi\left(M^2 + \sqrt{M^4 - J^2}\right) = \frac{4\pi J}{M\Omega}, \tag{7}$$

which implies that under variations $M \to M + \delta M$ and $J \to J + \delta J$ one has

$$\delta A = 16\pi\left(M + \frac{M^3}{\sqrt{M^4-J^2}}\right)(\delta M - \Omega\delta J)\,. \tag{8}$$

Therefore, Hawking's area law $\delta A \geq 0$ expressed for a Kerr BH is equivalent to $\delta M - \Omega\delta J \geq 0$.

Now suppose that we are interested in a scalar field propagating in the Kerr spacetime. Because of the isometries encoded in $\xi_{(t)}$ and $\xi_{(\phi)}$, we can expand the scalar field φ in modes:

$$\varphi(t,r,\theta,\phi) = f(r,\theta) \times \exp\left(-i\omega t + im\phi\right). \tag{9}$$

The energy and angular momentum of a quantum of that scalar field are $\epsilon = \hbar\omega$ and $j = \hbar m$. If this quantum falls into the BH, one has $\delta J/\delta M = m/\omega$. Then, however, the Hawking area theorem implies

$$\delta M - \Omega\delta J \geq 0 \quad \Leftrightarrow \quad \frac{\delta M}{\hbar\omega}\left(\hbar\omega - \hbar m\Omega\right) \geq 0 \tag{10}$$

Note that $\hbar\omega < \hbar m\Omega$ implies $\delta M < 0$. Therefore, energy is extracted from the BH. Due to the stationary nature of the Kerr spacetime, the frequency of the particle remains the same. In the language of field theory, thus, the amplitude of the field has to increase. In terms of quantum field theory and second quantization: a new particle is produced. This effect constitutes superradiance around rotating BHs.

2 Light bosons in particle physics

Dimensional estimates for a bosonic particle of mass μ let us conclude that it only forms bound states around a BH, if its Compton wavelength is comparable to the gravitational radius:

$$\lambda = \frac{h}{\mu c} \sim r_h \sim \frac{GM}{c^2}. \tag{11}$$

Typical astrophysical black holes have a mass of $M \sim \mathcal{O}\left(1\ldots10M_\odot\right)$ equivalent to a gravitational radius of $\mathcal{O}\left(1\ldots15\text{km}\right)$, which is a huge Compton wavelength amounting to the mass

$$\mu \sim \frac{hc}{MG} \sim 1.48\times10^{-45}\left(\frac{M_\odot}{M}\right)\text{ kg} \sim 0.83\times10^{-9}\left(\frac{M_\odot}{M}\right)\text{ eV}. \tag{12}$$

For astrophysical BHs, this mass is much smaller than the hypothesized neutrino masses of $\mathcal{O}(0.001\text{eV})$ implied by neutrino oscillations and cosmological estimates.

How does such a boson fit into the standard model? Why is it well-motivated to look for such a particle? This is the question we would like to answer next.

The strong CP problem

Unlike the weak interaction, which couples to chiral currents $\overline{\Psi}_\text{L}\gamma^\mu\Psi_\text{L}$, the strong interaction in quantum chromodynamics (QCD) couples to vector currents $\overline{\Psi}_\text{D}\gamma^\mu\Psi_\text{D}$, where Ψ_D denotes Dirac spinors containing quark fields [11]. Since a CP transformations reverses the space (sometimes also called a "mirror transformation") and charge-conjugates the particles, a left-handed particle gets transformed into a right-handed particle, and vice versa. Consequently, there are no CP-violating pieces allowed in the electroweak Lagrangian, since it couples exclusively to left-handed particles or right-handed antiparticles.

In the QCD Lagrangian, however, these pieces might be allowed, because QCD is not a chiral theory. However, it is an experimental fact that there are no CP-violating interactions detected in QCD, even though it is not required by any fundamental symmetry. So the question is: why is there no CP violation in QCD, if CP-violating terms are allowed to appear in the QCD Lagrangian? To that end, a possibly CP-violating QCD Lagrangian can be written as

$$\mathscr{L}_{\mathrm{QCD}}^{\not{\mathrm{CP}}} = -\frac{1}{2}F \wedge \star F - \frac{\theta}{16\pi^2}F \wedge F + \overline{\Psi}_{\mathrm{D}}\left(i\gamma \cdot \mathrm{D} - m\right)\Psi_{\mathrm{D}}, \tag{13}$$

where we suppressed the SU(3) Lie algebra traces. The CP-violating term is the 4-form $F \wedge F$ which can only be used in the Lagrangian in four dimensions. One way to explain the absence of these terms is to promote θ to a pseudoscalar field: then the theory is symmetric under $\theta \to \theta + \mathrm{const}$ because $F \wedge F = \mathrm{d}\left(F \wedge A + \frac{1}{3}A \wedge A \wedge A\right)$. Adding kinetic terms as well as potential break that symmetry and let this field acquire a very small vacuum expectation value and a very small mass: then, $\theta \lll 1$, and the CP-violating terms nearly drop out of the Lagrangian.

The first to introduce this kind of mechanism were Peccei and Quinn [12], and other proposals followed [13, 14, 15]. The Peccei–Quinn axion mass has to be of the order of $10^{-3}\,\mathrm{eV}$, but there are other models which feature different mass regimes. As an aside, legend has it that the name "axion" was conceived by Wilczek, who in turn was inspired by the name of a detergent he saw in a commercial.

At any rate, these mechanisms provide an explanation as to why QCD is CP invariant. Nevertheless, the particles have to be detected in order to confirm these theories. Merely employing standard model interactions (that is, looking for the axion at particle colliders) proves very difficult because of the very small mass. Already for neutrinos, large underwater tanks of water are needed to visualize the secondary Cherenkov radiation by cascaded muons.

But as we have seen: astrophysical BHs have just the right size to probe mass regimes as that of the axion. In what follows, we will study how the axion and other light bosons interact with gravity.

3 The gravitational atom

Before we move on to what is called the "gravitational atom," we should recall what constitutes an atom that is held together by electromagnetic forces. Quantum mechanically, an atom corresponds to a bound state of a positively charged nucleus surrounded by negatively charged electron states. The electrons interact under exchange of photons and may occupy different energy levels labeled by the quantum numbers n, l, and m. Due to the Pauli exclusion principle, there can only be two electrons with the same quantum numbers, provided they have different spin. Electrons can be excited by absorb a photon, which in turn leads to the electron to jump to the next free energy level. Conversely, electrons can emit a photon and jump to an energy level with lower energy. Semiclassically, the electrons move on orbits of quantized angular momentum.

For the gravitational atom, there are several parallels. An axion can occupy various bound states around a BH, and the stabilizing effect is provided by the gravitational attraction of the light, but still massive particles onto the central BH. The bound states, as we will see, are again labeled by integers, with the striking difference that now one can have an arbitrarily large amount of particles in a single bound state because axions are bosons. The interaction between the (pseudo-)scalar axions is either direct (via self-interaction terms in the Lagrangian), or indirect via the exchange of gravitons: These Feynman rules can be read off from the term $h_{\mu\nu}T^{\mu\nu}$, which describes the coupling of the graviton to the scalar particle. Similarly to the case of the hydrogen atom, axions can be excited to a higher energy orbit by absorption of a graviton, and they can emit a graviton and and jump to a lower energy orbit. Moreover, and this is a feature of scalar particles, two axions can annihilate each other and produce two gravitons. (They can also produce one graviton, because the kinematically forbidden process in flat spacetime is possible in curved spacetime due to the presence of the BH.) In what follows, we would like to make this more precise by following closely Arvanitaki *et al.* [16, 17, 18].

Let us now consider a scalar field in the Kerr geometry and derive the superradiance condition more rigorously. These concepts are explained in Arvanitaki and Dubovsky [16] and we will follow them closely here. The scalar Lagrangian and energy momentum tensor are

$$\mathscr{L} = -\frac{1}{2}\left(\partial_\mu\varphi\right)\left(\partial^\mu\varphi\right) - \frac{1}{2}\mu^2\varphi^2, \tag{14}$$

$$T_{\mu\nu} = \frac{1}{2}g_{\mu\nu}\left(\partial_\rho\varphi\right)\left(\partial^\rho\varphi\right) - \frac{1}{2}g_{\mu\nu}\mu^2\varphi^2 - \left(\partial_\mu\varphi\right)\left(\partial_\nu\varphi\right). \tag{15}$$

Parametrizing the field as $\varphi = \exp(-i\omega t + im\phi)f(r,\theta) + \text{h.c.}$ one can calculate the energy flux vector $P_\mu := -T_{\mu\nu}\xi^\nu_{(t)}$ obtains for the flux through the horizon (ℓ^μ is the null generator)

$$-T_{\mu\nu}\xi^\mu_{(t)}\ell^\nu = \omega\left(\omega - m\Omega\right)|f|^2. \tag{16}$$

Notably, in the domain of SR this becomes negative. One can now show that particles in the SR regime have (quasi-)non-relativistic velocities, and the real part of the frequency has the following hydrogen-like form:

$$\omega_{\overline{n}} \approx \mu\left(1 - \frac{\alpha^2}{2\overline{n}^2}\right), \quad \overline{n} := n + l + 1, \tag{17}$$

where $\overline{n}$ denotes the principal quantum number, l is the orbital angular momentum quantum number, and we defined the "gravitational fine structure constant" $\alpha := \mu GM$. The velocity of a particle on such an orbit is $v \sim \alpha/\overline{n}$, and the approximation $\omega_{\overline{n}} \approx \mu$ inserted into the SR condition (10) gives the estimate $\alpha \lesssim m\Omega = ma/(2r_h)$ such that

$$v \lesssim \frac{ma}{2\overline{n}r_h} < \frac{1}{2}, \tag{18}$$

which is quasi non-relativistic. According to [16], the corresponding radius of the "axion cloud" is then given by

$$r_c \sim \frac{\overline{n}^2}{\alpha^2} GM. \tag{19}$$

In the SR regime (10), the cloud is well away from the horizon, $r_c > r_h$. Numerical and semi-analytic methods developed in [16] (see also references therein) suggest that the hydrogenic approximation is justified to some extent. The intuitive picture is the following: the axion cloud (one for each set of quantum numbers) is localized around r_c and orbits the central BH on a Keplerian orbit. However, the axion cloud has a tail which probes the near-horizon region of the BH, and this is where the process of superradiant amplification takes place.

In general, the frequency of a wave in vicinity of a BH has a non-vanishing imaginary part due to the presence of absorption and SR. So far we only discussed the real part of the frequency. The imaginary part of the frequency, however, is what determines the growth rate of a mode. It can be shown that it takes the following approximate form:

$$\Gamma_{lmn} = 2\mu\alpha^{4l+4} r_h \left(m\Omega - \mu\right) C_{lmn} =: \Gamma_{\mathrm{sr}}, \tag{20}$$

where C_{lmn} is some rather involved expression containing the quantum numbers n, l, and m. Within our non-relativistic approximation we assumed $\omega \approx \mu$ (see above), so we can reproduce the negative sign of the imaginary part of the frequency in the domain of SR, leading to an amplification of modes. There are also alternative approaches to find approximate solutions, including the WKB method, but we will not discuss this further.

Dynamical processes

As we have seen, SR can enhance amplitudes provided the SR condition (10) is satisfied. But what happens next? There are two competing effects: (i) on the one hand, the ever-growing particle number in the first energy level extracts mass and angular momentum from the BH. This leads to a decrease of the parameters J and M and hence affects the SR condition, and at some point it might no longer be satisfied. (ii) On the other hand, the increasing particle number will lead to more and more self-interactions, which might become relevant at some point.

In what follows, we will mostly give qualitative arguments for relevant physical processes. Numerical derivations and more details can be found in [16, 17, 18].

Free field The imaginary part of the frequency is sometimes also referred to as the *SR rate*, because it has units of an inverse time. For astrophysical BHs and typical axion masses, the characteristic time scale is $\Gamma_{\rm sr}^{-1} \sim 10^{-3}\,$yrs, which is much shorter than, say, the Eddington accretion time of $\mathcal{O}(10^8\,\text{yrs})$ and the spinning-up time of a BH to a high value of $J/m^2 \sim 0.98$ [7]. That is to say, SR will proceed to happen as long as (i) its timescale remains smaller than the Eddington accretion time, and (ii) the SR condition (10) has not yet been saturated. During that time, the occupation number in a given state grows exponentially such that

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \Gamma_{\rm sr} N. \tag{21}$$

When enough angular momentum has been extracted, and the SR condition is no longer valid, one has the following approximate number of axions:

$$N_{\rm max} \simeq \frac{GM^2}{m}\Delta a_* \sim 10^{76}\left(\frac{\Delta a_*}{0.1}\right)\left(\frac{M}{10M_\odot}\right)^2, \tag{22}$$

where $\Delta a_* := \Delta a/M$ is the difference of the BH spin parameter before and after SR divided my the mass.

Level transitions It is possible for axions to absorb a graviton and change their energy level. These processes are only relevant when the axion clouds are large and the transition rate for a single axion becomes comparable to the growth rate per particle in the axion cloud. There are two different scenarios:

If at the onset of level transitions the excited state is growing faster than the ground state, the excited level becomes depleted until both the ground state and the excited state are occupied by an equal amount of particles. If, however, the ground state is growing faster than the excited state, the excited level becomes depleted even faster.

Bosenova If one takes axion self-interactions into account, there is a critical occupation number for which the axion cloud collapses and the hydrogen-like description is no longer valid. This phenomenon is sometimes referred to as a "bosenova." An estimate for this number is given by

$$N_{\rm bosenova} \simeq 10^{78}\frac{n^4}{\alpha^3}\left(\frac{M}{10M_\odot}\right)^2\left(\frac{f_a}{M_{\rm Pl}}\right)^2, \tag{23}$$

where n is the orbit quantum number, $M_{\rm Pl} \simeq 10^{18}\,$GeV is the Planck mass, and f_a is the decay constant of the axion. One can insert the above into the estimate (22) and convert it into an estimate for the axion mass. One finds that the bosenova is only relevant for axion masses $\mu \gtrsim 10^{-10}\,$eV, which is larger than the masses we are interested in in this review. Moreover, note that the bosenova collapses can be periodic events, until the SR condition is saturated.

Boson-boson-graviton annihilation Another effect of self-interactions is the following: suppose SR has successfully populated an energy level l, and extracted some angular momentum such that the SR condition is no longer valid. Then, SR would begin filling the next level, $l+1$, but this is only true if self-interactions are neglected. In case of non-trivial self-interactions (which is a reasonable assumption for the QCD axion), these will lead to a mixing of superradiant energy levels l with non-superradiant levels with different quantum numbers: two bosons will annihilate to one graviton. The time scale of these self-interactions is set by the details of the theory, but it can be much longer than the SR timescale. As soon as the level l is depleted to the point that this level-mixing is no longer relevant, the next level $l+1$ will start to be populated by the mechanism of SR.

Boson annihilation with BH background gravitons The bosons can of course also annihilate with gravitons stemming from the background BH. We will discuss the observational significance further below.

Complete saturation Eventually, there will be no next quantum number that still saturates the SR condition, at which point the entire process will come to an end. The BH will locate a permanent spot in the (M, J) diagram, the precise position of which might give some hints on the axion energy levels and mass. Then, annihilation is the predominant process.

4 Observational signatures

All of the mentioned processes have observational signatures, which we would like to address as a final point.

Absence of fast-spinning BHs The field of gravitational wave astronomy is still very young, so at the present time it is difficult to predict the influence it will have on future science. Assuming that more BH mergers will be detected, each discovery will provide at least the pair (m, J) for each BH, which can be used for filling in the (m, J) diagram. If there is a notable and statistically significant absence of fast-spinning BHs in that diagram, it might be an indication for the presence of some light boson. Extracting the boson's parameters solely from these diagrams is possible, but it relies on the knowledge of the statistical distribution of the BH masses and spins [18]. The leading uncertainty is given by the BH masses, since the masses show up in higher polynomial orders such that their relative error multiplies.

A more direct evidence for the existence of a light boson stems from gravitational wave (GW) emission due to either boson level transitions, boson-boson annihilations, or bosenovas.

Gravitational waves – transitions and annihilations Although there was some dispute in the literature, it has been claimed [17] that gravitational wave emission originating from transitions and annihilations between bosons is monochromatic, that is, has a fixed frequency

$$\omega_{\overline{n}} \approx \frac{1}{2}\mu\alpha^2\left(\frac{1}{\overline{n}_g^2} - \frac{1}{\overline{n}_e^2}\right), \tag{24}$$

where $\overline{n}_g$ and $\overline{n}_e$ denote the quantum numbers of the ground state and the excited state. Disregarding angular orientation, the strain of a gravitational wave of frequency ω and power P emitted at distance r away from Earth can be estimated to

$$h = \sqrt{\frac{4GP}{r^2\omega^2}} \tag{25}$$

For the emission of level transitions one can estimate $P \sim \Gamma_t N_g(t) N_e(t)$, where g and e denote the ground state and the excited state, respectively, and Γ_t is the transition rate for a single axion that can be estimated numerically.

Level transitions (for $\Gamma_e > \Gamma_g$) are characterized by a growing GW strain until the excited level has the same number of particles as the ground state, which is when the GW signal drops quickly. For $\Gamma_g > \Gamma_e$, however, the GW strain is suppressed all the time since the transitions to the ground state are almost negligible.

For both of these processes, the relevant timescale is on the order of decades—much longer than the science runs at LIGO or VIRGO, as the authors of [17] point out. The fact that these signlas have not been observed yet is due to the relatively small strain of e.g. $h \sim 10^{-24}$ for a BH which is 10kpc away. This is noticeably smaller than the GW150914 strain of appr. 10^{-21}.

Annihilations will set in as soon as the SR condition (10) is no longer satisfied. If the number of particles in a given state is still low enough such that the self-interactions will not trigger the bosenova process, the GW strain is unaffected.

Gravitational waves – annihilations with BH gravitons If the axion-BH system only features one relevant energy level, there is a competition between SR amplification of that energy level and graviton annihilation of the same level caused by gravitons emitted by the BH. Being an annihilation frequency, it is roughly given by

$$\omega \approx 2\mu, \tag{26}$$

making this frequency very distinctive. It can be shown that this process is by far the slowest: it is much slower than the time scale of level transitions, which in turn is much slower than SR. If the system is dominated by one single energy level, the annihilation signal can last for thousands of years. The power of emitted GW in this case can be estimated to $P \sim N^2(t)\Gamma_a$, where Γ_a is the annihilation rate, and $N(t)$ is the number of particles after the SR condition is no longer saturated, that it, it is a decreasing function in time. The signal strain is comparable to a transition signal for a BH of a distance of 10kpc, $h \sim 10^{-24}$.

Gravitational waves – bosenova The bosenova process has a different observational signature: the GW emission need not be monochromatic, and usually there are several bosenovae following one another in a given system. For typical astrophysical BHs, there will be $\mathcal{O}(10)$ GW bursts lasting a few milliseconds, separated by intervals of a few minutes. The strain of these events, again at a sample distance of 10 kpc, is slightly larger, $h \sim 10^{-21}$.

Conclusions

We have seen that the presence of a light boson around a rotating BH may trigger the mechanism of SR. In consequence, the processes of level transitions, annihilations, bosenovae, or annihilations with external gravitons will leave distinct signatures in form of gravitational waves, whose strain is still just beyond the grasp of experimental validation.

In our review we focused on the phenomenological aspects and did not discuss refinements of the model as elucidated in the literature: numerical and semi-analytical methods as well as statistical analysis of BH parameter abundances can sharpen the predictions to some extent, a discussion of which would be beyond the scope of this term paper.

What does the future hold? Provided more BH merger events are detected, the (M, J) diagram may be populated with more data points, allowing us to determine whether there is an absence of fast spinning BHs or not. Further improvements on ground-based GW observatories, as well as the possibility of space-based observatories as well as other new methods [19] may yield more insights and might open the window to smaller GW strains. On the other hand, advances in particle physics might provide experimentalists with different or more detailed observational signatures. Either way, it seems like the Pandora's gravitational wave box has only just been opened.

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